Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.0-a-cos-^m-b-trg-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [294]. This is test number [82].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (294)	% 0. (0)
Mathematica	% 100. (294)	% 0. (0)
Maple	% 66.33 (195)	% 33.67 (99)
Maxima	% 31.29 (92)	% 68.71 (202)
Fricas	% 31.63 (93)	% 68.37 (201)
Sympy	% 4.42 (13)	% 95.58 (281)
Giac	% 9.18 (27)	% 90.82 (267)

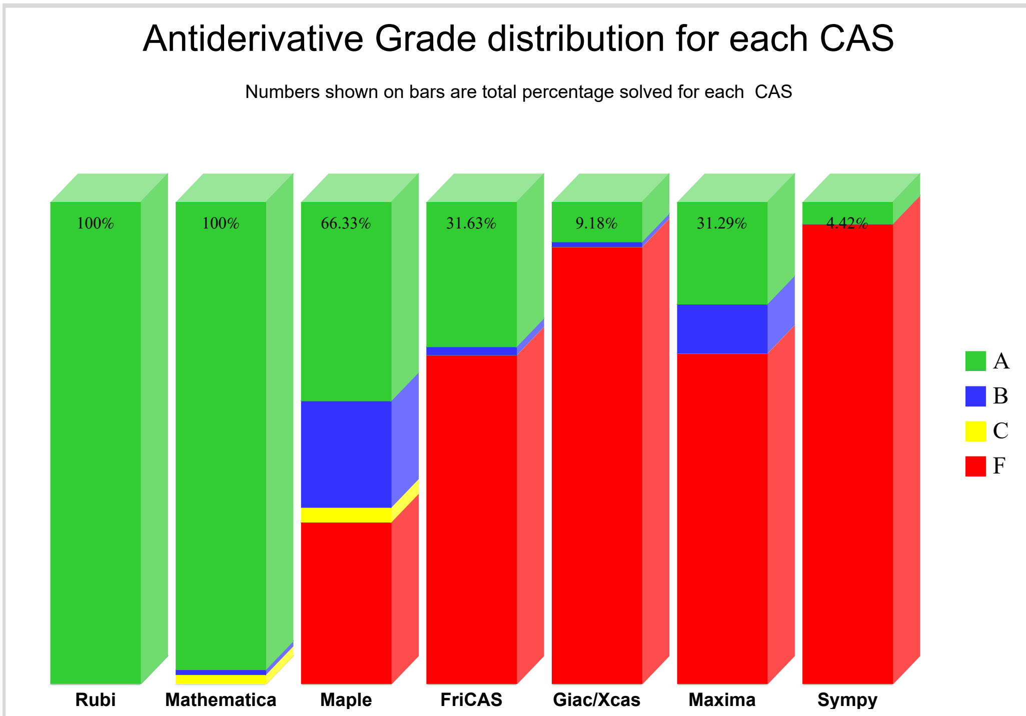
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

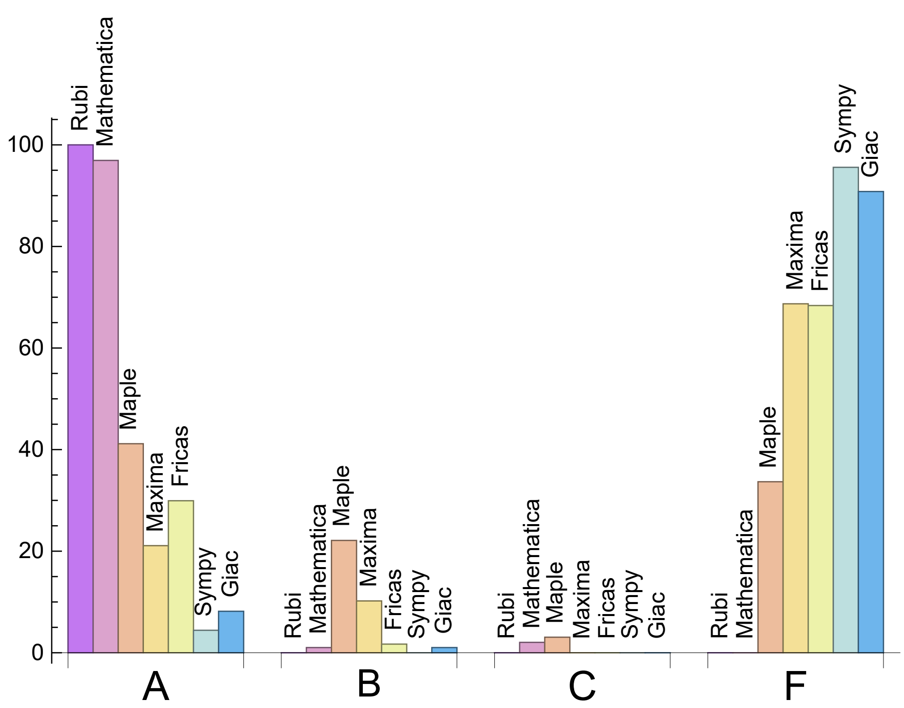
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	96.94	1.02	2.04	0.
Maple	41.16	22.11	3.06	33.67
Maxima	21.09	10.2	0.	68.71
Fricas	29.93	1.7	0.	68.37
Sympy	4.42	0.	0.	95.58
Giac	8.16	1.02	0.	90.82

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.04	68.33	1.	69.	1.
Mathematica	0.19	59.7	0.91	55.5	0.91
Maple	1.14	140.95	1.98	121.	1.67
Maxima	1.79	298.59	3.79	65.5	1.18
Fricas	1.73	246.58	4.5	144.	2.71
Sympy	4.39	59.31	1.46	46.	1.44
Giac	1.65	60.48	1.76	43.	1.09

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {284, 285, 286, 287, 292}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

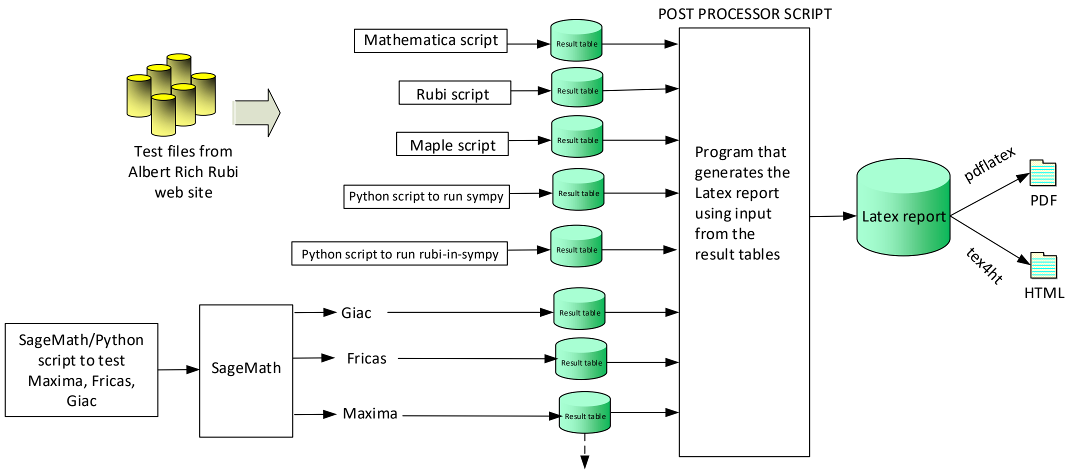
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 288, 289, 290, 291, 293, 294 }

B grade: { 1, 42, 43 }

C grade: { 276, 284, 285, 286, 287, 292 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 14, 17, 22, 39, 40, 41, 44, 51, 52, 53, 54, 55, 56, 66, 68, 73, 78, 80, 85, 90, 92, 97, 102, 103, 104, 105, 110, 115, 116, 117, 122, 127, 128, 129, 134, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278 }

B grade: { 9, 10, 11, 12, 15, 16, 18, 19, 20, 23, 24, 42, 43, 67, 69, 70, 71, 72, 74, 75, 76, 77, 79, 81, 82, 83, 84, 86, 87, 88, 89, 91, 93, 94, 95, 96, 98, 99, 100, 101, 106, 107, 108, 111, 112, 113, 114, 118, 119, 120, 121, 123, 124, 125, 126, 130, 131, 132, 133, 135, 136, 137, 138, 139, 274 }

C grade: { 13, 21, 45, 46, 47, 48, 49, 50, 109 }

F grade: { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 57, 58, 59, 60, 61, 62, 63, 64, 65, 156, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 51, 52, 53, 54, 55, 56, 140, 141, 142, 143, 144, 146, 150, 151, 152, 153, 154, 156, 160, 161, 162, 163, 164, 165, 167, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 192, 193, 194, 195, 196, 261, 262, 265, 266, 267, 270, 271, 272, 275, 276

B grade: { 42, 43, 44, 145, 147, 148, 149, 155, 157, 158, 159, 166, 168, 169, 170, 177, 178, 179, 180, 181, 187, 188, 189, 190, 191, 197, 198, 199, 200, 201 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 268, 269, 273, 274, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 43, 44, 51, 52, 53, 54, 55, 56, 64, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 261, 262, 265, 266, 267, 270 }

B grade: { 42, 271, 272, 275, 276 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 268, 269, 273, 274, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 41, 64, 143, 144, 176 }

B grade: { }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 43, 51, 52, 53, 56, 143, 261, 265, 266, 267, 270, 271, 275 }

B grade: { 44, 64, 262 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 42, 45, 46, 47, 48, 49, 50, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 268, 269, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	14	22	12	14
normalized size	1	1.	2.1	1.1	1.4	2.2	1.2	1.4
time (sec)	N/A	0.004	0.008	0.022	1.187	1.335	0.124	1.177

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	30	55	46	24
normalized size	1	1.	0.92	1.08	1.2	2.2	1.84	0.96
time (sec)	N/A	0.009	0.022	0.026	1.318	1.36	0.218	1.2

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	30	55	36	30
normalized size	1	1.	1.	0.85	1.15	2.12	1.38	1.15
time (sec)	N/A	0.011	0.008	0.028	1.337	1.574	0.476	1.329

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	38	45	89	95	43
normalized size	1	1.	0.72	0.83	0.98	1.93	2.07	0.93
time (sec)	N/A	0.021	0.038	0.027	1.386	1.648	1.012	1.299

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	46	85	58	46
normalized size	1	1.	1.	0.78	1.12	2.07	1.41	1.12
time (sec)	N/A	0.013	0.014	0.027	1.49	1.642	2.004	1.261

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	48	65	120	139	62
normalized size	1	1.	0.64	0.72	0.97	1.79	2.07	0.93
time (sec)	N/A	0.033	0.038	0.027	1.443	1.641	3.827	1.188

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	42	59	112	78	59
normalized size	1	1.	1.	0.78	1.09	2.07	1.44	1.09
time (sec)	N/A	0.016	0.013	0.026	1.639	1.689	6.884	1.519

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	55	58	80	153	184	81
normalized size	1	1.	0.62	0.66	0.91	1.74	2.09	0.92
time (sec)	N/A	0.047	0.055	0.026	1.635	1.439	12.022	1.386

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	199	0	0	0	0
normalized size	1	1.	0.78	3.06	0.	0.	0.	0.
time (sec)	N/A	0.032	0.11	2.068	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	40	202	0	0	0	0
normalized size	1	1.	0.95	4.81	0.	0.	0.	0.
time (sec)	N/A	0.018	0.05	1.908	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	179	0	0	0	0
normalized size	1	1.	0.86	4.26	0.	0.	0.	0.
time (sec)	N/A	0.018	0.043	1.506	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	133	0	0	0	0
normalized size	1	1.	1.	8.31	0.	0.	0.	0.
time (sec)	N/A	0.009	0.019	1.359	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	0	0	0	0
normalized size	1	1.	1.	1.12	0.	0.	0.	0.
time (sec)	N/A	0.009	0.031	0.037	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	101	0	0	0	0
normalized size	1	1.	1.	2.66	0.	0.	0.	0.
time (sec)	N/A	0.018	0.053	1.684	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	213	0	0	0	0
normalized size	1	1.	0.86	5.07	0.	0.	0.	0.
time (sec)	N/A	0.018	0.06	2.024	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	358	0	0	0	0
normalized size	1	1.	0.91	5.51	0.	0.	0.	0.
time (sec)	N/A	0.029	0.093	3.014	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	76	210	0	0	0	0
normalized size	1	1.	0.78	2.14	0.	0.	0.	0.
time (sec)	N/A	0.059	0.092	2.047	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	62	213	0	0	0	0
normalized size	1	1.	0.89	3.04	0.	0.	0.	0.
time (sec)	N/A	0.036	0.084	1.855	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	58	190	0	0	0	0
normalized size	1	1.	0.83	2.71	0.	0.	0.	0.
time (sec)	N/A	0.037	0.046	1.884	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	142	0	0	0	0
normalized size	1	1.	1.	3.74	0.	0.	0.	0.
time (sec)	N/A	0.022	0.019	1.457	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	54	0	0	0	0
normalized size	1	1.	1.	1.42	0.	0.	0.	0.
time (sec)	N/A	0.023	0.025	0.204	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	168	0	0	0	0
normalized size	1	1.	0.74	2.47	0.	0.	0.	0.
time (sec)	N/A	0.036	0.03	2.045	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	241	0	0	0	0
normalized size	1	1.	0.71	3.35	0.	0.	0.	0.
time (sec)	N/A	0.035	0.059	1.763	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	68	366	0	0	0	0
normalized size	1	1.	0.68	3.66	0.	0.	0.	0.
time (sec)	N/A	0.056	0.095	3.175	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.058	0.112	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.033	0.167	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.029	0.157	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.025	0.151	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.025	0.139	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.026	0.111	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.049	0.114	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.038	0.191	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.037	0.167	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.041	0.158	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.044	0.155	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.041	0.115	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.046	0.368	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	64	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.044	0.447	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	42	107	0	46
normalized size	1	1.	0.68	0.6	0.79	2.02	0.	0.87
time (sec)	N/A	0.039	0.019	0.628	2.347	1.123	0.	1.403

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	23	74	0	23
normalized size	1	1.	0.76	0.71	0.68	2.18	0.	0.68
time (sec)	N/A	0.027	0.009	0.605	2.559	1.046	0.	1.417

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	8	43	19	12
normalized size	1	1.	1.	1.15	0.62	3.31	1.46	0.92
time (sec)	N/A	0.011	0.004	0.337	2.322	1.071	0.577	1.232

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	46	48	51	182	0	0
normalized size	1	1.	2.88	3.	3.19	11.38	0.	0.
time (sec)	N/A	0.014	0.018	0.708	2.676	1.178	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	91	70	410	124	0	59
normalized size	1	1.	2.17	1.67	9.76	2.95	0.	1.4
time (sec)	N/A	0.022	0.064	1.105	2.33	1.145	0.	1.789

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	72	89	1260	151	0	170
normalized size	1	1.	1.18	1.46	20.66	2.48	0.	2.79
time (sec)	N/A	0.044	0.157	1.112	3.661	1.146	0.	1.569

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	61	114	0	0	0	0
normalized size	1	1.	0.52	0.97	0.	0.	0.	0.
time (sec)	N/A	0.066	0.12	0.301	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	50	198	0	0	0	0
normalized size	1	1.	0.75	2.96	0.	0.	0.	0.
time (sec)	N/A	0.041	0.065	0.254	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	37	76	0	0	0	0
normalized size	1	1.	0.84	1.73	0.	0.	0.	0.
time (sec)	N/A	0.028	0.025	0.262	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	191	0	0	0	0
normalized size	1	1.	0.74	4.55	0.	0.	0.	0.
time (sec)	N/A	0.023	0.021	0.333	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	44	87	0	0	0	0
normalized size	1	1.	0.62	1.23	0.	0.	0.	0.
time (sec)	N/A	0.03	0.059	0.289	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	57	223	0	0	0	0
normalized size	1	1.	0.49	1.91	0.	0.	0.	0.
time (sec)	N/A	0.051	0.105	0.423	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	53	57	115	200	0	77
normalized size	1	1.	0.4	0.43	0.87	1.52	0.	0.58
time (sec)	N/A	0.051	0.123	0.295	2.473	1.161	0.	1.399

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	38	41	74	128	0	34
normalized size	1	1.	0.49	0.53	0.95	1.64	0.	0.44
time (sec)	N/A	0.032	0.069	0.162	2.312	1.081	0.	1.427

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	25	22	30	69	0	18
normalized size	1	1.	0.69	0.61	0.83	1.92	0.	0.5
time (sec)	N/A	0.015	0.014	0.192	2.428	1.049	0.	1.557

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	8	51	0	0
normalized size	1	1.	1.	0.93	0.53	3.4	0.	0.
time (sec)	N/A	0.014	0.006	0.181	2.059	1.028	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	30	29	30	101	0	0
normalized size	1	1.	0.45	0.43	0.45	1.51	0.	0.
time (sec)	N/A	0.021	0.028	0.114	1.899	1.042	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	47	41	46	147	0	46
normalized size	1	1.	0.4	0.35	0.39	1.26	0.	0.39
time (sec)	N/A	0.032	0.052	0.165	1.883	1.085	0.	1.303

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.061	0.274	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.173	0.419	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.119	0.2	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	68	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.07	0.239	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.075	0.2	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	72	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.106	0.192	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.106	0.197	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	32	65	405
normalized size	1	1.	1.	0.	0.	1.33	2.71	16.88
time (sec)	N/A	0.02	0.03	0.26	0.	1.049	1.542	8.073

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	74	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.057	0.289	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	83	234	0	0	0	0
normalized size	1	1.	0.67	1.9	0.	0.	0.	0.
time (sec)	N/A	0.09	0.22	1.885	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	75	221	0	0	0	0
normalized size	1	1.	0.77	2.28	0.	0.	0.	0.
time (sec)	N/A	0.069	0.154	1.934	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	73	208	0	0	0	0
normalized size	1	1.	0.77	2.19	0.	0.	0.	0.
time (sec)	N/A	0.066	0.145	2.051	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	62	211	0	0	0	0
normalized size	1	1.	0.9	3.06	0.	0.	0.	0.
time (sec)	N/A	0.041	0.062	1.817	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	61	188	0	0	0	0
normalized size	1	1.	0.91	2.81	0.	0.	0.	0.
time (sec)	N/A	0.043	0.064	1.864	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	142	0	0	0	0
normalized size	1	1.	1.	3.74	0.	0.	0.	0.
time (sec)	N/A	0.02	0.023	1.457	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	142	0	0	0	0
normalized size	1	1.	1.	3.64	0.	0.	0.	0.
time (sec)	N/A	0.029	0.031	1.5	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	48	166	0	0	0	0
normalized size	1	1.	0.76	2.63	0.	0.	0.	0.
time (sec)	N/A	0.055	0.054	1.892	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	49	239	0	0	0	0
normalized size	1	1.	0.7	3.41	0.	0.	0.	0.
time (sec)	N/A	0.057	0.078	1.944	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	69	363	0	0	0	0
normalized size	1	1.	0.73	3.82	0.	0.	0.	0.
time (sec)	N/A	0.073	0.233	2.887	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	69	396	0	0	0	0
normalized size	1	1.	0.7	4.04	0.	0.	0.	0.
time (sec)	N/A	0.072	0.194	1.804	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	79	412	0	0	0	0
normalized size	1	1.	0.64	3.35	0.	0.	0.	0.
time (sec)	N/A	0.097	0.211	3.457	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	83	236	0	0	0	0
normalized size	1	1.	0.66	1.87	0.	0.	0.	0.
time (sec)	N/A	0.086	0.2	1.842	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	223	0	0	0	0
normalized size	1	1.	0.79	2.35	0.	0.	0.	0.
time (sec)	N/A	0.06	0.1	1.695	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	73	210	0	0	0	0
normalized size	1	1.	0.74	2.14	0.	0.	0.	0.
time (sec)	N/A	0.061	0.1	1.868	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	65	213	0	0	0	0
normalized size	1	1.	0.97	3.18	0.	0.	0.	0.
time (sec)	N/A	0.039	0.098	1.779	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	58	190	0	0	0	0
normalized size	1	1.	0.83	2.71	0.	0.	0.	0.
time (sec)	N/A	0.033	0.015	2.003	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	144	0	0	0	0
normalized size	1	1.	1.	3.69	0.	0.	0.	0.
time (sec)	N/A	0.029	0.015	1.563	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	0	0	0
normalized size	1	1.	1.	3.51	0.	0.	0.	0.
time (sec)	N/A	0.038	0.024	1.359	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	168	0	0	0	0
normalized size	1	1.	0.76	2.55	0.	0.	0.	0.
time (sec)	N/A	0.054	0.056	1.958	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	241	0	0	0	0
normalized size	1	1.	0.71	3.35	0.	0.	0.	0.
time (sec)	N/A	0.054	0.068	2.011	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	69	364	0	0	0	0
normalized size	1	1.	0.7	3.71	0.	0.	0.	0.
time (sec)	N/A	0.074	0.125	3.349	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	69	398	0	0	0	0
normalized size	1	1.	0.69	3.98	0.	0.	0.	0.
time (sec)	N/A	0.073	0.129	1.856	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	79	414	0	0	0	0
normalized size	1	1.	0.63	3.29	0.	0.	0.	0.
time (sec)	N/A	0.095	0.19	3.475	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	83	236	0	0	0	0
normalized size	1	1.	0.66	1.89	0.	0.	0.	0.
time (sec)	N/A	0.08	0.195	2.044	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	75	223	0	0	0	0
normalized size	1	1.	0.77	2.28	0.	0.	0.	0.
time (sec)	N/A	0.059	0.101	1.743	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	76	210	0	0	0	0
normalized size	1	1.	0.78	2.16	0.	0.	0.	0.
time (sec)	N/A	0.058	0.095	2.015	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	62	213	0	0	0	0
normalized size	1	1.	0.89	3.04	0.	0.	0.	0.
time (sec)	N/A	0.033	0.059	1.883	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	190	0	0	0	0
normalized size	1	1.	0.82	2.64	0.	0.	0.	0.
time (sec)	N/A	0.048	0.024	1.786	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	0	0	0
normalized size	1	1.	1.	3.51	0.	0.	0.	0.
time (sec)	N/A	0.039	0.015	1.441	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	144	0	0	0	0
normalized size	1	1.	0.93	3.51	0.	0.	0.	0.
time (sec)	N/A	0.038	0.044	1.421	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	168	0	0	0	0
normalized size	1	1.	0.74	2.47	0.	0.	0.	0.
time (sec)	N/A	0.055	0.058	2.019	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	241	0	0	0	0
normalized size	1	1.	0.71	3.35	0.	0.	0.	0.
time (sec)	N/A	0.056	0.07	1.871	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	69	366	0	0	0	0
normalized size	1	1.	0.69	3.66	0.	0.	0.	0.
time (sec)	N/A	0.073	0.157	3.274	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	69	398	0	0	0	0
normalized size	1	1.	0.69	3.98	0.	0.	0.	0.
time (sec)	N/A	0.074	0.178	1.819	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	79	414	0	0	0	0
normalized size	1	1.	0.62	3.23	0.	0.	0.	0.
time (sec)	N/A	0.095	0.26	3.356	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	76	210	0	0	0	0
normalized size	1	1.	0.78	2.14	0.	0.	0.	0.
time (sec)	N/A	0.051	0.021	1.878	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	73	233	0	0	0	0
normalized size	1	1.	0.58	1.86	0.	0.	0.	0.
time (sec)	N/A	0.091	0.142	2.155	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	71	220	0	0	0	0
normalized size	1	1.	0.71	2.2	0.	0.	0.	0.
time (sec)	N/A	0.059	0.123	1.894	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	63	207	0	0	0	0
normalized size	1	1.	0.65	2.13	0.	0.	0.	0.
time (sec)	N/A	0.062	0.105	1.7	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	210	0	0	0	0
normalized size	1	1.	0.81	2.92	0.	0.	0.	0.
time (sec)	N/A	0.039	0.056	1.682	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	187	0	0	0	0
normalized size	1	1.	0.74	2.71	0.	0.	0.	0.
time (sec)	N/A	0.039	0.045	1.863	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	141	0	0	0	0
normalized size	1	1.	1.	3.44	0.	0.	0.	0.
time (sec)	N/A	0.023	0.015	1.409	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	54	0	0	0	0
normalized size	1	1.	1.	1.42	0.	0.	0.	0.
time (sec)	N/A	0.019	0.015	0.2	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	47	165	0	0	0	0
normalized size	1	1.	0.72	2.54	0.	0.	0.	0.
time (sec)	N/A	0.044	0.043	2.019	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	48	238	0	0	0	0
normalized size	1	1.	0.72	3.55	0.	0.	0.	0.
time (sec)	N/A	0.051	0.064	1.994	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	65	366	0	0	0	0
normalized size	1	1.	0.67	3.77	0.	0.	0.	0.
time (sec)	N/A	0.073	0.098	3.485	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	63	395	0	0	0	0
normalized size	1	1.	0.66	4.16	0.	0.	0.	0.
time (sec)	N/A	0.072	0.144	1.967	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	77	411	0	0	0	0
normalized size	1	1.	0.62	3.29	0.	0.	0.	0.
time (sec)	N/A	0.094	0.315	3.677	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	76	236	0	0	0	0
normalized size	1	1.	0.59	1.84	0.	0.	0.	0.
time (sec)	N/A	0.083	0.115	2.177	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	74	223	0	0	0	0
normalized size	1	1.	0.74	2.23	0.	0.	0.	0.
time (sec)	N/A	0.058	0.113	1.93	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	66	210	0	0	0	0
normalized size	1	1.	0.66	2.1	0.	0.	0.	0.
time (sec)	N/A	0.06	0.082	1.99	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	61	213	0	0	0	0
normalized size	1	1.	0.85	2.96	0.	0.	0.	0.
time (sec)	N/A	0.04	0.052	2.039	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	54	190	0	0	0	0
normalized size	1	1.	0.75	2.64	0.	0.	0.	0.
time (sec)	N/A	0.041	0.042	1.93	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	0	0	0
normalized size	1	1.	1.	3.51	0.	0.	0.	0.
time (sec)	N/A	0.023	0.023	1.325	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	0	0	0
normalized size	1	1.	1.	3.51	0.	0.	0.	0.
time (sec)	N/A	0.024	0.02	1.46	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	168	0	0	0	0
normalized size	1	1.	0.74	2.47	0.	0.	0.	0.
time (sec)	N/A	0.034	0.028	1.992	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	241	0	0	0	0
normalized size	1	1.	0.74	3.49	0.	0.	0.	0.
time (sec)	N/A	0.048	0.026	1.894	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	68	366	0	0	0	0
normalized size	1	1.	0.69	3.73	0.	0.	0.	0.
time (sec)	N/A	0.076	0.081	3.25	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	66	398	0	0	0	0
normalized size	1	1.	0.68	4.1	0.	0.	0.	0.
time (sec)	N/A	0.075	0.089	2.064	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	80	414	0	0	0	0
normalized size	1	1.	0.63	3.29	0.	0.	0.	0.
time (sec)	N/A	0.096	0.181	3.647	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	76	236	0	0	0	0
normalized size	1	1.	0.59	1.84	0.	0.	0.	0.
time (sec)	N/A	0.082	0.114	2.044	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	74	223	0	0	0	0
normalized size	1	1.	0.74	2.23	0.	0.	0.	0.
time (sec)	N/A	0.06	0.098	2.151	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	66	210	0	0	0	0
normalized size	1	1.	0.66	2.1	0.	0.	0.	0.
time (sec)	N/A	0.06	0.054	2.086	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	61	213	0	0	0	0
normalized size	1	1.	0.85	2.96	0.	0.	0.	0.
time (sec)	N/A	0.04	0.045	2.027	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	54	190	0	0	0	0
normalized size	1	1.	0.75	2.64	0.	0.	0.	0.
time (sec)	N/A	0.041	0.043	1.984	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	0	0	0
normalized size	1	1.	1.	3.51	0.	0.	0.	0.
time (sec)	N/A	0.023	0.015	1.608	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	144	0	0	0	0
normalized size	1	1.	0.93	3.51	0.	0.	0.	0.
time (sec)	N/A	0.026	0.059	1.586	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	168	0	0	0	0
normalized size	1	1.	0.74	2.47	0.	0.	0.	0.
time (sec)	N/A	0.042	0.038	2.231	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	241	0	0	0	0
normalized size	1	1.	0.71	3.35	0.	0.	0.	0.
time (sec)	N/A	0.035	0.018	1.909	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	68	366	0	0	0	0
normalized size	1	1.	0.7	3.77	0.	0.	0.	0.
time (sec)	N/A	0.07	0.06	3.269	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	66	398	0	0	0	0
normalized size	1	1.	0.67	4.06	0.	0.	0.	0.
time (sec)	N/A	0.079	0.071	1.937	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	80	414	0	0	0	0
normalized size	1	1.	0.64	3.31	0.	0.	0.	0.
time (sec)	N/A	0.098	0.089	3.252	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	68	366	0	0	0	0
normalized size	1	1.	0.68	3.66	0.	0.	0.	0.
time (sec)	N/A	0.053	0.018	3.435	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	55	62	66	495	0	0
normalized size	1	1.	0.56	0.63	0.67	5.05	0.	0.
time (sec)	N/A	0.027	0.095	0.316	1.818	1.938	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	40	57	112	0	0
normalized size	1	1.	0.64	0.57	0.81	1.6	0.	0.
time (sec)	N/A	0.017	0.089	0.283	1.828	1.707	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	45	42	34	427	0	0
normalized size	1	1.	0.71	0.67	0.54	6.78	0.	0.
time (sec)	N/A	0.014	0.055	0.273	1.851	1.899	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	18	78	29	42
normalized size	1	1.	1.	0.91	0.56	2.44	0.91	1.31
time (sec)	N/A	0.007	0.028	0.269	1.78	1.612	20.833	3.377

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	28	35	265	5	0
normalized size	1	1.	1.	1.17	1.46	11.04	0.21	0.
time (sec)	N/A	0.003	0.011	0.188	1.542	1.866	3.874	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	88	311	0	0
normalized size	1	1.	1.	1.27	2.67	9.42	0.	0.
time (sec)	N/A	0.008	0.012	0.268	1.822	1.841	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	73	78	0	0
normalized size	1	1.	1.	0.91	2.28	2.44	0.	0.
time (sec)	N/A	0.012	0.021	0.271	1.765	1.625	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	52	104	892	559	0	0
normalized size	1	1.	0.72	1.44	12.39	7.76	0.	0.
time (sec)	N/A	0.021	0.047	0.289	1.838	1.885	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	42	397	115	0	0
normalized size	1	1.	0.64	0.6	5.67	1.64	0.	0.
time (sec)	N/A	0.018	0.085	0.269	1.84	1.639	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	66	121	2236	628	0	0
normalized size	1	1.	0.62	1.13	20.9	5.87	0.	0.
time (sec)	N/A	0.036	0.106	0.306	2.08	1.924	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	55	62	72	509	0	0
normalized size	1	1.	0.54	0.61	0.71	5.04	0.	0.
time (sec)	N/A	0.029	0.089	0.184	1.827	1.933	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	45	40	61	117	0	0
normalized size	1	1.	0.62	0.56	0.85	1.62	0.	0.
time (sec)	N/A	0.018	0.117	0.175	1.815	1.679	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	42	38	435	0	0
normalized size	1	1.	0.69	0.65	0.58	6.69	0.	0.
time (sec)	N/A	0.015	0.055	0.25	1.787	1.95	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	18	81	0	0
normalized size	1	1.	0.97	0.88	0.55	2.45	0.	0.
time (sec)	N/A	0.007	0.044	0.224	1.77	1.589	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	28	35	267	0	0
normalized size	1	1.	0.96	1.12	1.4	10.68	0.	0.
time (sec)	N/A	0.003	0.016	0.145	1.544	1.879	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	42	92	313	0	0
normalized size	1	1.	0.97	1.24	2.71	9.21	0.	0.
time (sec)	N/A	0.008	0.016	0.162	1.827	1.936	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	0	73	81	0	0
normalized size	1	1.	0.97	0.	2.21	2.45	0.	0.
time (sec)	N/A	0.012	0.023	180.	1.806	1.612	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	104	933	567	0	0
normalized size	1	1.	0.7	1.41	12.61	7.66	0.	0.
time (sec)	N/A	0.021	0.054	0.17	1.889	1.932	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	45	42	404	117	0	0
normalized size	1	1.	0.62	0.58	5.61	1.62	0.	0.
time (sec)	N/A	0.018	0.06	0.165	1.829	1.609	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	67	121	2352	641	0	0
normalized size	1	1.	0.61	1.1	21.38	5.83	0.	0.
time (sec)	N/A	0.039	0.089	0.195	2.197	2.155	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	57	52	104	158	0	0
normalized size	1	1.	0.49	0.45	0.9	1.36	0.	0.
time (sec)	N/A	0.026	0.125	0.169	1.867	1.976	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	55	62	80	522	0	0
normalized size	1	1.	0.51	0.58	0.75	4.88	0.	0.
time (sec)	N/A	0.03	0.085	0.175	1.833	2.295	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	40	66	123	0	0
normalized size	1	1.	0.59	0.53	0.87	1.62	0.	0.
time (sec)	N/A	0.018	0.144	0.243	1.816	1.987	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	42	43	443	0	0
normalized size	1	1.	0.65	0.61	0.62	6.42	0.	0.
time (sec)	N/A	0.016	0.072	0.234	1.829	2.193	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	18	84	0	0
normalized size	1	1.	0.91	0.83	0.51	2.4	0.	0.
time (sec)	N/A	0.008	0.053	0.154	1.77	1.864	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	28	35	270	0	0
normalized size	1	1.	0.89	1.04	1.3	10.	0.	0.
time (sec)	N/A	0.003	0.012	0.131	1.538	2.151	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	42	97	316	0	0
normalized size	1	1.	0.92	1.17	2.69	8.78	0.	0.
time (sec)	N/A	0.009	0.022	0.145	1.845	2.121	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	73	84	0	0
normalized size	1	1.	0.91	0.83	2.09	2.4	0.	0.
time (sec)	N/A	0.012	0.02	0.154	1.795	1.933	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	52	104	1008	575	0	0
normalized size	1	1.	0.67	1.33	12.92	7.37	0.	0.
time (sec)	N/A	0.021	0.06	0.162	1.851	2.183	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	42	420	123	0	0
normalized size	1	1.	0.59	0.55	5.53	1.62	0.	0.
time (sec)	N/A	0.019	0.059	0.159	1.88	1.852	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	66	121	2584	655	0	0
normalized size	1	1.	0.57	1.04	22.28	5.65	0.	0.
time (sec)	N/A	0.036	0.116	0.192	2.219	2.198	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	57	52	92	144	0	0
normalized size	1	1.	0.53	0.49	0.86	1.35	0.	0.
time (sec)	N/A	0.024	0.114	0.247	1.941	2.016	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	55	62	66	506	0	0
normalized size	1	1.	0.56	0.63	0.67	5.16	0.	0.
time (sec)	N/A	0.027	0.074	0.257	1.825	2.291	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	40	57	115	0	0
normalized size	1	1.	0.64	0.57	0.81	1.64	0.	0.
time (sec)	N/A	0.017	0.081	0.257	1.774	1.949	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	45	42	34	437	0	0
normalized size	1	1.	0.71	0.67	0.54	6.94	0.	0.
time (sec)	N/A	0.015	0.054	0.244	1.766	2.25	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	18	81	0	0
normalized size	1	1.	1.	0.91	0.56	2.53	0.	0.
time (sec)	N/A	0.007	0.031	0.246	1.785	1.824	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	28	35	274	5	0
normalized size	1	1.	1.	1.17	1.46	11.42	0.21	0.
time (sec)	N/A	0.003	0.011	0.168	1.568	2.251	3.721	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	88	319	0	0
normalized size	1	1.	1.	1.27	2.67	9.67	0.	0.
time (sec)	N/A	0.007	0.012	0.229	1.815	2.124	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	80	81	0	0
normalized size	1	1.	1.	0.91	2.5	2.53	0.	0.
time (sec)	N/A	0.012	0.023	0.238	1.769	1.818	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	52	104	892	564	0	0
normalized size	1	1.	0.72	1.44	12.39	7.83	0.	0.
time (sec)	N/A	0.022	0.036	0.25	1.826	2.128	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	42	397	117	0	0
normalized size	1	1.	0.64	0.6	5.67	1.67	0.	0.
time (sec)	N/A	0.018	0.076	0.251	1.817	1.904	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	66	121	2236	633	0	0
normalized size	1	1.	0.62	1.13	20.9	5.92	0.	0.
time (sec)	N/A	0.035	0.079	0.282	1.89	2.235	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	55	62	66	512	0	0
normalized size	1	1.	0.51	0.58	0.62	4.79	0.	0.
time (sec)	N/A	0.029	0.084	0.178	1.865	2.32	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	40	57	117	0	0
normalized size	1	1.	0.59	0.53	0.75	1.54	0.	0.
time (sec)	N/A	0.019	0.067	0.157	1.817	1.899	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	42	34	443	0	0
normalized size	1	1.	0.65	0.61	0.49	6.42	0.	0.
time (sec)	N/A	0.016	0.039	0.161	1.786	2.265	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	18	84	0	0
normalized size	1	1.	0.91	0.83	0.51	2.4	0.	0.
time (sec)	N/A	0.007	0.041	0.158	1.746	1.832	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	28	35	277	0	0
normalized size	1	1.	0.89	1.04	1.3	10.26	0.	0.
time (sec)	N/A	0.003	0.016	0.145	1.556	2.246	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	42	88	321	0	0
normalized size	1	1.	0.92	1.17	2.44	8.92	0.	0.
time (sec)	N/A	0.008	0.013	0.23	1.786	2.192	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	90	84	0	0
normalized size	1	1.	0.91	0.83	2.57	2.4	0.	0.
time (sec)	N/A	0.013	0.015	0.233	1.763	1.859	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	52	104	905	570	0	0
normalized size	1	1.	0.67	1.33	11.6	7.31	0.	0.
time (sec)	N/A	0.022	0.037	0.167	1.851	2.295	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	42	420	120	0	0
normalized size	1	1.	0.59	0.55	5.53	1.58	0.	0.
time (sec)	N/A	0.019	0.026	0.158	1.805	1.958	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	66	121	2267	639	0	0
normalized size	1	1.	0.57	1.04	19.54	5.51	0.	0.
time (sec)	N/A	0.037	0.055	0.191	1.914	2.39	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	58	62	66	512	0	0
normalized size	1	1.	0.54	0.58	0.62	4.79	0.	0.
time (sec)	N/A	0.028	0.088	0.178	1.812	2.662	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	48	40	57	117	0	0
normalized size	1	1.	0.63	0.53	0.75	1.54	0.	0.
time (sec)	N/A	0.018	0.065	0.155	1.825	2.208	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	48	42	34	443	0	0
normalized size	1	1.	0.7	0.61	0.49	6.42	0.	0.
time (sec)	N/A	0.015	0.044	0.158	1.779	2.394	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	29	18	84	0	0
normalized size	1	1.	1.	0.83	0.51	2.4	0.	0.
time (sec)	N/A	0.007	0.026	0.152	1.792	1.682	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	28	35	277	0	0
normalized size	1	1.	0.89	1.04	1.3	10.26	0.	0.
time (sec)	N/A	0.003	0.012	0.136	1.579	1.855	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	42	88	321	0	0
normalized size	1	1.	0.92	1.17	2.44	8.92	0.	0.
time (sec)	N/A	0.008	0.025	0.157	1.791	1.85	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	90	84	0	0
normalized size	1	1.	0.91	0.83	2.57	2.4	0.	0.
time (sec)	N/A	0.013	0.017	0.237	1.742	1.666	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	55	102	929	570	0	0
normalized size	1	1.	0.71	1.31	11.91	7.31	0.	0.
time (sec)	N/A	0.022	0.046	0.245	1.861	1.919	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	42	463	120	0	0
normalized size	1	1.	0.59	0.55	6.09	1.58	0.	0.
time (sec)	N/A	0.018	0.028	0.161	1.876	1.662	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	66	121	2334	639	0	0
normalized size	1	1.	0.57	1.04	20.12	5.51	0.	0.
time (sec)	N/A	0.037	0.066	0.191	1.925	1.986	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.108	0.141	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.062	0.213	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.053	0.146	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.036	0.232	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.048	0.181	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.059	0.147	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.048	0.161	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.117	0.135	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.061	0.177	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.054	0.131	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.037	0.198	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.046	0.177	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.059	0.143	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.043	0.154	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.178	0.139	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.08	0.177	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.074	0.13	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.003	0.118	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.023	0.157	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	56	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.028	0.182	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.061	0.155	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.115	0.137	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.076	0.165	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.005	0.177	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.003	0.171	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.049	0.144	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	58	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.056	0.148	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.109	0.157	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.113	0.138	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.076	0.16	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.02	0.161	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.023	0.146	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.048	0.135	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	58	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.058	0.142	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.103	0.152	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.11	0.129	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.023	0.197	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.008	0.171	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.022	0.111	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.013	0.135	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	58	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.063	0.145	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.11	0.155	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	77	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.085	1.096	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	72	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.078	1.323	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	70	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.054	1.167	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	64	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.042	0.398	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	63	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.056	0.711	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	67	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.065	0.658	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.047	0.769	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	72	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.056	0.57	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.14	0.241	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.119	0.223	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.093	0.241	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.059	0.22	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.082	0.218	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.087	0.232	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.088	0.229	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.09	0.227	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	10.494	1.065	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	32	0	27
normalized size	1	1.	1.	0.93	1.2	2.13	0.	1.8
time (sec)	N/A	0.025	0.021	0.042	0.938	1.107	0.	1.142

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	68	0	43
normalized size	1	1.	1.	0.82	1.06	4.	0.	2.53
time (sec)	N/A	0.025	0.024	0.029	0.943	1.097	0.	1.31

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	53	88	0	0	0	0
normalized size	1	1.	0.79	1.31	0.	0.	0.	0.
time (sec)	N/A	0.051	0.101	1.098	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	61	142	0	0	0	0
normalized size	1	1.	0.91	2.12	0.	0.	0.	0.
time (sec)	N/A	0.048	0.139	1.122	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	18	72	0	23
normalized size	1	1.	0.86	0.67	0.86	3.43	0.	1.1
time (sec)	N/A	0.024	0.025	0.51	0.963	1.022	0.	1.131

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	26	34	63	0	42
normalized size	1	1.	0.82	0.79	1.03	1.91	0.	1.27
time (sec)	N/A	0.033	0.058	0.565	0.957	1.071	0.	1.183

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	29	26	34	95	0	34
normalized size	1	1.	0.83	0.74	0.97	2.71	0.	0.97
time (sec)	N/A	0.033	0.064	0.53	0.998	1.107	0.	1.253

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	63	100	0	0	0	0
normalized size	1	1.	0.68	1.09	0.	0.	0.	0.
time (sec)	N/A	0.08	0.147	1.016	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	63	152	0	0	0	0
normalized size	1	1.	0.68	1.65	0.	0.	0.	0.
time (sec)	N/A	0.079	0.412	1.095	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	26	0	8
normalized size	1	1.	1.	0.7	0.8	2.6	0.	0.8
time (sec)	N/A	0.017	0.008	0.026	0.963	1.041	0.	1.149

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	47	28	55	274	0	66
normalized size	1	1.	1.47	0.88	1.72	8.56	0.	2.06
time (sec)	N/A	0.028	0.026	0.471	1.507	1.279	0.	1.207

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	50	48	55	275	0	0
normalized size	1	1.	1.61	1.55	1.77	8.87	0.	0.
time (sec)	N/A	0.029	0.034	1.172	1.487	1.315	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	49	123	0	0	0	0
normalized size	1	1.	0.8	2.02	0.	0.	0.	0.
time (sec)	N/A	0.048	0.148	1.664	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	177	0	0	0	0
normalized size	1	1.	0.87	2.85	0.	0.	0.	0.
time (sec)	N/A	0.048	0.148	1.586	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	73	73	88	373	0	99
normalized size	1	1.	1.18	1.18	1.42	6.02	0.	1.6
time (sec)	N/A	0.053	0.114	1.51	1.441	1.292	0.	1.262

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	33	71	85	400	0	0
normalized size	1	1.	0.53	1.15	1.37	6.45	0.	0.
time (sec)	N/A	0.046	0.031	1.461	1.455	1.303	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	64	168	0	0	0	0
normalized size	1	1.	0.7	1.83	0.	0.	0.	0.
time (sec)	N/A	0.081	0.378	1.954	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	76	160	0	0	0	0
normalized size	1	1.	0.83	1.74	0.	0.	0.	0.
time (sec)	N/A	0.08	0.242	2.687	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	105	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.641	0.279	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.171	0.31	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	68	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.216	0.299	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	68	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.274	0.283	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.286	0.286	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	312	0	0	0	0	0
normalized size	1	1.	3.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	1.885	0.927	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	314	0	0	0	0	0
normalized size	1	1.	3.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.3	1.004	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	314	0	0	0	0	0
normalized size	1	1.	3.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.271	0.98	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	316	0	0	0	0	0
normalized size	1	1.	3.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.275	1.109	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	76	94	0	0	0	0	0
normalized size	1	0.97	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	7.907	0.361	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	94	0	0	0	0	0
normalized size	1	1.	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	2.451	0.333	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	94	0	0	0	0	0
normalized size	1	1.	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	1.257	0.33	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	96	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	1.08	0.374	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	225	0	0	0	0	0
normalized size	1	1.	2.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	1.714	0.365	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	116	0	0	0	0	0
normalized size	1	1.	1.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	6.77	0.309	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	125	0	0	0	0	0
normalized size	1	1.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	1.114	0.318	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [275] had the largest ratio of [0.3158]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	6	0.167
2	A	2	2	1.	8	0.25
3	A	2	1	1.	8	0.125
4	A	3	2	1.	8	0.25
5	A	2	1	1.	8	0.125
6	A	4	2	1.	8	0.25
7	A	2	1	1.	8	0.125
8	A	5	2	1.	8	0.25
9	A	3	2	1.	10	0.2
10	A	2	2	1.	10	0.2
11	A	2	2	1.	10	0.2
12	A	1	1	1.	10	0.1
13	A	1	1	1.	10	0.1
14	A	2	2	1.	10	0.2
15	A	2	2	1.	10	0.2
16	A	3	2	1.	10	0.2
17	A	4	3	1.	12	0.25
18	A	3	3	1.	12	0.25
19	A	3	3	1.	12	0.25
20	A	2	2	1.	12	0.167
21	A	2	2	1.	12	0.167
22	A	3	3	1.	12	0.25
23	A	3	3	1.	12	0.25
24	A	4	3	1.	12	0.25
25	A	1	1	1.	10	0.1
26	A	1	1	1.	10	0.1
27	A	1	1	1.	10	0.1
28	A	1	1	1.	10	0.1
29	A	1	1	1.	10	0.1
30	A	1	1	1.	10	0.1
31	A	1	1	1.	12	0.083
32	A	1	1	1.	12	0.083
33	A	1	1	1.	12	0.083
34	A	1	1	1.	12	0.083
35	A	1	1	1.	12	0.083

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	1	1	1.	12	0.083
37	A	1	1	1.	8	0.125
38	A	1	1	1.	10	0.1
39	A	4	3	1.	10	0.3
40	A	3	3	1.	10	0.3
41	A	2	2	1.	10	0.2
42	A	2	2	1.	10	0.2
43	A	3	3	1.	10	0.3
44	A	4	3	1.	10	0.3
45	A	6	3	1.	10	0.3
46	A	4	3	1.	10	0.3
47	A	3	3	1.	10	0.3
48	A	3	3	1.	10	0.3
49	A	4	3	1.	10	0.3
50	A	6	3	1.	10	0.3
51	A	7	3	1.	10	0.3
52	A	5	3	1.	10	0.3
53	A	3	3	1.	10	0.3
54	A	3	3	1.	10	0.3
55	A	3	2	1.	10	0.2
56	A	3	2	1.	10	0.2
57	A	2	2	1.	12	0.167
58	A	2	2	1.	14	0.143
59	A	2	2	1.	14	0.143
60	A	2	2	1.	14	0.143
61	A	2	2	1.	14	0.143
62	A	2	2	1.	14	0.143
63	A	2	2	1.	14	0.143
64	A	2	2	1.	14	0.143
65	A	2	2	1.	14	0.143
66	A	6	4	1.	21	0.19
67	A	5	4	1.	21	0.19
68	A	5	4	1.	21	0.19
69	A	4	4	1.	21	0.19
70	A	4	4	1.	19	0.21
71	A	2	2	1.	12	0.167
72	A	3	3	1.	19	0.158
73	A	4	4	1.	21	0.19
74	A	4	4	1.	21	0.19
75	A	5	4	1.	21	0.19
76	A	5	4	1.	21	0.19
77	A	6	4	1.	21	0.19
78	A	6	4	1.	21	0.19
79	A	5	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	5	4	1.	21	0.19
81	A	4	4	1.	19	0.21
82	A	3	3	1.	12	0.25
83	A	3	3	1.	19	0.158
84	A	3	3	1.	21	0.143
85	A	4	4	1.	21	0.19
86	A	4	4	1.	21	0.19
87	A	5	4	1.	21	0.19
88	A	5	4	1.	21	0.19
89	A	6	4	1.	21	0.19
90	A	6	4	1.	21	0.19
91	A	5	4	1.	21	0.19
92	A	5	4	1.	19	0.21
93	A	3	3	1.	12	0.25
94	A	4	4	1.	19	0.21
95	A	3	3	1.	21	0.143
96	A	3	3	1.	21	0.143
97	A	4	4	1.	21	0.19
98	A	4	4	1.	21	0.19
99	A	5	4	1.	21	0.19
100	A	5	4	1.	21	0.19
101	A	6	4	1.	21	0.19
102	A	4	3	1.	12	0.25
103	A	6	4	1.	21	0.19
104	A	5	4	1.	21	0.19
105	A	5	4	1.	21	0.19
106	A	4	4	1.	21	0.19
107	A	4	4	1.	21	0.19
108	A	3	3	1.	19	0.158
109	A	2	2	1.	12	0.167
110	A	4	4	1.	19	0.21
111	A	4	4	1.	21	0.19
112	A	5	4	1.	21	0.19
113	A	5	4	1.	21	0.19
114	A	6	4	1.	21	0.19
115	A	6	4	1.	21	0.19
116	A	5	4	1.	21	0.19
117	A	5	4	1.	21	0.19
118	A	4	4	1.	21	0.19
119	A	4	4	1.	21	0.19
120	A	3	3	1.	21	0.143
121	A	3	3	1.	19	0.158
122	A	3	3	1.	12	0.25
123	A	4	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	5	4	1.	21	0.19
125	A	5	4	1.	21	0.19
126	A	6	4	1.	21	0.19
127	A	6	4	1.	21	0.19
128	A	5	4	1.	21	0.19
129	A	5	4	1.	21	0.19
130	A	4	4	1.	21	0.19
131	A	4	4	1.	21	0.19
132	A	3	3	1.	21	0.143
133	A	3	3	1.	21	0.143
134	A	4	4	1.	19	0.21
135	A	3	3	1.	12	0.25
136	A	5	4	1.	19	0.21
137	A	5	4	1.	21	0.19
138	A	6	4	1.	21	0.19
139	A	4	3	1.	12	0.25
140	A	4	3	1.	23	0.13
141	A	3	2	1.	23	0.087
142	A	3	3	1.	23	0.13
143	A	2	2	1.	23	0.087
144	A	2	2	1.	23	0.087
145	A	2	2	1.	23	0.087
146	A	3	3	1.	23	0.13
147	A	3	3	1.	23	0.13
148	A	3	2	1.	23	0.087
149	A	4	3	1.	23	0.13
150	A	4	3	1.	23	0.13
151	A	3	2	1.	23	0.087
152	A	3	3	1.	23	0.13
153	A	2	2	1.	23	0.087
154	A	2	2	1.	23	0.087
155	A	2	2	1.	23	0.087
156	A	3	3	1.	23	0.13
157	A	3	3	1.	23	0.13
158	A	3	2	1.	23	0.087
159	A	4	3	1.	23	0.13
160	A	3	2	1.	23	0.087
161	A	4	3	1.	23	0.13
162	A	3	2	1.	23	0.087
163	A	3	3	1.	23	0.13
164	A	2	2	1.	23	0.087
165	A	2	2	1.	23	0.087
166	A	2	2	1.	23	0.087
167	A	3	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
168	A	3	3	1.	23	0.13
169	A	3	2	1.	23	0.087
170	A	4	3	1.	23	0.13
171	A	3	2	1.	23	0.087
172	A	4	3	1.	23	0.13
173	A	3	2	1.	23	0.087
174	A	3	3	1.	23	0.13
175	A	2	2	1.	23	0.087
176	A	2	2	1.	23	0.087
177	A	2	2	1.	23	0.087
178	A	3	3	1.	23	0.13
179	A	3	3	1.	23	0.13
180	A	3	2	1.	23	0.087
181	A	4	3	1.	23	0.13
182	A	4	3	1.	23	0.13
183	A	3	2	1.	23	0.087
184	A	3	3	1.	23	0.13
185	A	2	2	1.	23	0.087
186	A	2	2	1.	23	0.087
187	A	2	2	1.	23	0.087
188	A	3	3	1.	23	0.13
189	A	3	3	1.	23	0.13
190	A	3	2	1.	23	0.087
191	A	4	3	1.	23	0.13
192	A	4	3	1.	23	0.13
193	A	3	2	1.	23	0.087
194	A	3	3	1.	23	0.13
195	A	2	2	1.	23	0.087
196	A	2	2	1.	23	0.087
197	A	2	2	1.	23	0.087
198	A	3	3	1.	23	0.13
199	A	3	3	1.	23	0.13
200	A	3	2	1.	23	0.087
201	A	4	3	1.	23	0.13
202	A	2	2	1.	21	0.095
203	A	2	2	1.	21	0.095
204	A	2	2	1.	19	0.105
205	A	1	1	1.	12	0.083
206	A	2	2	1.	19	0.105
207	A	2	2	1.	21	0.095
208	A	2	2	1.	21	0.095
209	A	2	2	1.	21	0.095
210	A	2	2	1.	21	0.095
211	A	2	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	A	1	1	1.	12	0.083
213	A	2	2	1.	19	0.105
214	A	2	2	1.	21	0.095
215	A	2	2	1.	21	0.095
216	A	2	2	1.	21	0.095
217	A	2	2	1.	21	0.095
218	A	2	2	1.	19	0.105
219	A	1	1	1.	12	0.083
220	A	2	2	1.	19	0.105
221	A	2	2	1.	21	0.095
222	A	2	2	1.	21	0.095
223	A	2	2	1.	21	0.095
224	A	2	2	1.	21	0.095
225	A	2	2	1.	19	0.105
226	A	1	1	1.	12	0.083
227	A	2	2	1.	19	0.105
228	A	2	2	1.	21	0.095
229	A	2	2	1.	21	0.095
230	A	2	2	1.	21	0.095
231	A	2	2	1.	21	0.095
232	A	2	2	1.	19	0.105
233	A	1	1	1.	12	0.083
234	A	2	2	1.	19	0.105
235	A	2	2	1.	21	0.095
236	A	2	2	1.	21	0.095
237	A	2	2	1.	21	0.095
238	A	2	2	1.	21	0.095
239	A	2	2	1.	19	0.105
240	A	1	1	1.	12	0.083
241	A	2	2	1.	19	0.105
242	A	2	2	1.	21	0.095
243	A	2	2	1.	21	0.095
244	A	2	2	1.	21	0.095
245	A	2	2	1.	19	0.105
246	A	2	2	1.	17	0.118
247	A	1	1	1.	10	0.1
248	A	2	2	1.	17	0.118
249	A	2	2	1.	19	0.105
250	A	2	2	1.	19	0.105
251	A	2	2	1.	19	0.105
252	A	2	2	1.	21	0.095
253	A	2	2	1.	21	0.095
254	A	2	2	1.	21	0.095
255	A	2	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	A	2	2	1.	21	0.095
257	A	2	2	1.	21	0.095
258	A	2	2	1.	21	0.095
259	A	2	2	1.	21	0.095
260	A	2	2	1.	21	0.095
261	A	2	2	1.	17	0.118
262	A	2	2	1.	17	0.118
263	A	3	3	1.	19	0.158
264	A	3	3	1.	19	0.158
265	A	3	2	1.	11	0.182
266	A	3	2	1.	19	0.105
267	A	3	2	1.	19	0.105
268	A	4	3	1.	19	0.158
269	A	4	3	1.	19	0.158
270	A	2	2	1.	9	0.222
271	A	5	5	1.	17	0.294
272	A	5	5	1.	17	0.294
273	A	3	3	1.	19	0.158
274	A	3	3	1.	19	0.158
275	A	6	6	1.	19	0.316
276	A	6	6	1.	19	0.316
277	A	4	3	1.	19	0.158
278	A	4	3	1.	19	0.158
279	A	2	2	1.	21	0.095
280	A	2	2	1.	21	0.095
281	A	2	2	1.	21	0.095
282	A	2	2	1.	21	0.095
283	A	2	2	1.	21	0.095
284	A	2	2	1.	17	0.118
285	A	2	2	1.	19	0.105
286	A	2	2	1.	19	0.105
287	A	2	2	1.	21	0.095
288	A	2	2	0.97	23	0.087
289	A	2	2	1.	23	0.087
290	A	2	2	1.	23	0.087
291	A	2	2	1.	23	0.087
292	A	2	2	1.	23	0.087
293	A	2	2	1.	23	0.087
294	A	2	2	1.	23	0.087

Chapter 3

Listing of integrals

3.1 $\int \cos(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\sin(a+bx)}{b}$$

```
[Out] Sin[a + b*x]/b
```

Rubi [A] time = 0.0041908, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2637}

$$\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x],x]
```

```
[Out] Sin[a + b*x]/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(a+bx)\,dx = \frac{\sin(a+bx)}{b}$$

Mathematica [B] time = 0.0083783, size = 21, normalized size = 2.1

$$\frac{\sin(a)\cos(bx)}{b}+\frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x],x]
```

```
[Out] (Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b
```

Maple [A] time = 0.022, size = 11, normalized size = 1.1

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a),x)
```

```
[Out] sin(b*x+a)/b
```

Maxima [A] time = 1.18738, size = 14, normalized size = 1.4

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="maxima")
```

```
[Out] sin(b*x + a)/b
```

Fricas [A] time = 1.33475, size = 22, normalized size = 2.2

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/b
```

Sympy [A] time = 0.124142, size = 12, normalized size = 1.2

$$\begin{cases} \frac{\sin(a+bx)}{b} & \text{for } b \neq 0 \\ x\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x)
```

```
[Out] Piecewise((sin(a + b*x)/b, Ne(b, 0)), (x*cos(a), True))
```

<u>Giac</u> [A] time = 1.17728, size = 14, normalized size = 1.4

$$\frac{\sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="giac")
```

```
[Out] sin(b*x + a)/b
```

3.2 $\int \cos^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

```
[Out] x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.0089483, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2,x]

[Out] x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\,dx &= \frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0219691, size = 23, normalized size = 0.92

$$\frac{2(a+bx)+\sin(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2,x]

[Out] (2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0.026, size = 27, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A] time = 1.31848, size = 30, normalized size = 1.2

$$\frac{2\,bx+2\,a+\sin(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))/b
```

Fricas [A] time = 1.36006, size = 55, normalized size = 2.2

$$\frac{bx+\cos(bx+a)\sin(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

Sympy [A] time = 0.218096, size = 46, normalized size = 1.84

$$\begin{cases}\frac{x\sin^2(a+bx)}{2}+\frac{x\cos^2(a+bx)}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b\neq 0\\ x\cos^2(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*cos(a)**2, True))
```

Giac [A] time = 1.19964, size = 24, normalized size = 0.96

$$\frac{1}{2}\,x+\frac{\sin(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*b*x + 2*a)/b
```

3.3 $\int \cos^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

```
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0112665, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3,x]

[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.007986, size = 26, normalized size = 1.

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3,x]

[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Maple [A] time = 0.028, size = 22, normalized size = 0.9

$$\frac{\left(2+(\cos(bx+a))^2\right)\sin(bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3,x)
```

```
[Out] 1/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A] time = 1.3374, size = 30, normalized size = 1.15

$$-\frac{\sin(bx+a)^3 - 3\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

Fricas [A] time = 1.57371, size = 55, normalized size = 2.12

$$\frac{\left(\cos(bx+a)^2 + 2\right)\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

Sympy [A] time = 0.475782, size = 36, normalized size = 1.38

$$\begin{cases} \frac{2\sin^3(a+bx)}{3b} + \frac{\sin(a+bx)\cos^2(a+bx)}{b} & \text{for } b \neq 0 \\ x\cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*sin(a + b*x)**3/(3*b) + sin(a + b*x)*cos(a + b*x)**2/b, Ne(b,
0)), (x*cos(a)**3, True))
```

Giac [A] time = 1.32938, size = 30, normalized size = 1.15

$$-\frac{\sin(bx+a)^3 - 3\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

3.4 $\int \cos^4(a + bx)\, dx$

Optimal. Leaf size=46

$$\frac{\sin(a + bx)\cos^3(a + bx)}{4b} + \frac{3\sin(a + bx)\cos(a + bx)}{8b} + \frac{3x}{8}$$

```
[Out] (3*x)/8 + (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) + (Cos[a + b*x]^3*Sin[a + b*x
])/(4*b)
```

Rubi [A] time = 0.0208342, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sin(a + bx)\cos^3(a + bx)}{4b} + \frac{3\sin(a + bx)\cos(a + bx)}{8b} + \frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) + (Cos[a + b*x]^3*Sin[a + b*x
])/(4*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(a + bx)\, dx &= \frac{\cos^3(a + bx)\sin(a + bx)}{4b} + \frac{3}{4}\int \cos^2(a + bx)\, dx \\ &= \frac{3\cos(a + bx)\sin(a + bx)}{8b} + \frac{\cos^3(a + bx)\sin(a + bx)}{4b} + \frac{3\int 1\, dx}{8} \\ &= \frac{3x}{8} + \frac{3\cos(a + bx)\sin(a + bx)}{8b} + \frac{\cos^3(a + bx)\sin(a + bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0380392, size = 33, normalized size = 0.72

$$\frac{12(a + bx) + 8\sin(2(a + bx)) + \sin(4(a + bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4,x]
```

```
[Out] (12*(a + b*x) + 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

Maple [A] time = 0.027, size = 38, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\sin(bx+a)}{4}\left((\cos(bx+a))^3+\frac{3\cos(bx+a)}{2}\right)+\frac{3bx}{8}+\frac{3a}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4,x)
```

```
[Out] 1/b*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)
```

Maxima [A] time = 1.38624, size = 45, normalized size = 0.98

$$\frac{12\,bx+12\,a+\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))/b
```

Fricas [A] time = 1.64788, size = 89, normalized size = 1.93

$$\frac{3\,bx+\left(2\,\cos(bx+a)^3+3\,\cos(bx+a)\right)\sin(bx+a)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*b*x + (2*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [A] time = 1.01157, size = 95, normalized size = 2.07

$$\begin{cases}\frac{3x\sin^4(a+bx)}{8}+\frac{3x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{3x\cos^4(a+bx)}{8}+\frac{3\sin^3(a+bx)\cos(a+bx)}{8b}+\frac{5\sin(a+bx)\cos^3(a+bx)}{8b} & \text{for } b\neq 0\\ x\cos^4(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
3*x*cos(a + b*x)**4/8 + 3*sin(a + b*x)**3*cos(a + b*x)/(8*b) + 5*sin(a + b*
x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*cos(a)**4, True))
```

Giac [A] time = 1.299, size = 43, normalized size = 0.93

$$\frac{3}{8}x + \frac{\sin\left(4\,bx + 4\,a\right)}{32\,b} + \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*b*x + 4*a)/b + 1/4*sin(2*b*x + 2*a)/b
```

3.5 $\int \cos^5(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\sin^5(a+bx)}{5b}-\frac{2\sin^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}$$

```
[Out] Sin[a + b*x]/b - (2*Sin[a + b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0129965, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\sin^5(a+bx)}{5b}-\frac{2\sin^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]/b - (2*Sin[a + b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(a+bx)\right)}{b}\\ &= \frac{\sin(a+bx)}{b}-\frac{2\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0139859, size = 41, normalized size = 1.

$$\frac{\sin^5(a+bx)}{5b}-\frac{2\sin^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]/b - (2*Sin[a + b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Maple [A] time = 0.027, size = 32, normalized size = 0.8

$$\frac{\sin(bx+a)}{5b}\left(\frac{8}{3}+(\cos(bx+a))^4+\frac{4\,(\cos(bx+a))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5,x)
```

```
[Out] 1/5/b*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A] time = 1.49008, size = 46, normalized size = 1.12

$$\frac{3\sin(bx+a)^5 - 10\sin(bx+a)^3 + 15\sin(bx+a)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(b*x + a)^5 - 10*sin(b*x + a)^3 + 15*sin(b*x + a))/b
```

Fricas [A] time = 1.64212, size = 85, normalized size = 2.07

$$\frac{\left(3\cos(bx+a)^4 + 4\cos(bx+a)^2 + 8\right)\sin(bx+a)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [A] time = 2.00431, size = 58, normalized size = 1.41

$$\begin{cases} \frac{8\sin^5(a+bx)}{15b} + \frac{4\sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{\sin(a+bx)\cos^4(a+bx)}{b} & \text{for } b \neq 0 \\ x\cos^5(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5,x)
```

```
[Out] Piecewise((8*sin(a + b*x)**5/(15*b) + 4*sin(a + b*x)**3*cos(a + b*x)**2/(3*
b) + sin(a + b*x)*cos(a + b*x)**4/b, Ne(b, 0)), (x*cos(a)**5, True))
```

Giac [A] time = 1.26137, size = 46, normalized size = 1.12

$$\frac{3\sin(bx+a)^5 - 10\sin(bx+a)^3 + 15\sin(bx+a)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(b*x + a)^5 - 10*sin(b*x + a)^3 + 15*sin(b*x + a))/b
```

3.6 $\int \cos^6(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{5\sin(a+bx)\cos(a+bx)}{16b}+\frac{5x}{16}$$

```
[Out] (5*x)/16 + (5*Cos[a + b*x]*Sin[a + b*x])/(16*b) + (5*Cos[a + b*x]^3*Sin[a +
 b*x])/(24*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(6*b)
```

Rubi [A] time = 0.0331819, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{5\sin(a+bx)\cos(a+bx)}{16b}+\frac{5x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[a + b*x]*Sin[a + b*x])/(16*b) + (5*Cos[a + b*x]^3*Sin[a +
 b*x])/(24*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(6*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(a+bx)\,dx&=\frac{\cos^5(a+bx)\sin(a+bx)}{6b}+\frac{5}{6}\int \cos^4(a+bx)\,dx\\&=\frac{5\cos^3(a+bx)\sin(a+bx)}{24b}+\frac{\cos^5(a+bx)\sin(a+bx)}{6b}+\frac{5}{8}\int \cos^2(a+bx)\,dx\\&=\frac{5\cos(a+bx)\sin(a+bx)}{16b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{24b}+\frac{\cos^5(a+bx)\sin(a+bx)}{6b}+\frac{5\int 1\,dx}{16}\\&=\frac{5x}{16}+\frac{5\cos(a+bx)\sin(a+bx)}{16b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{24b}+\frac{\cos^5(a+bx)\sin(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.0375434, size = 43, normalized size = 0.64

$$\frac{45\sin(2(a+bx))+9\sin(4(a+bx))+\sin(6(a+bx))+60a+60bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^6,x]
```

```
[Out] (60*a + 60*b*x + 45*Sin[2*(a + b*x)] + 9*Sin[4*(a + b*x)] + Sin[6*(a + b*x)
])/(192*b)
```

Maple [A] time = 0.027, size = 48, normalized size = 0.7

$$\frac{1}{b}\left(\frac{\sin(bx+a)}{6}\left((\cos(bx+a))^5+\frac{5(\cos(bx+a))^3}{4}+\frac{15\cos(bx+a)}{8}\right)+\frac{5bx}{16}+\frac{5a}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6,x)
```

```
[Out] 1/b*(1/6*(cos(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x+a))*sin(b*x+a)+5/16*b*
x+5/16*a)
```

Maxima [A] time = 1.44311, size = 65, normalized size = 0.97

$$-\frac{4\sin(2bx+2a)^3-60bx-60a-9\sin(4bx+4a)-48\sin(2bx+2a)}{192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/192*(4*sin(2*b*x + 2*a)^3 - 60*b*x - 60*a - 9*sin(4*b*x + 4*a) - 48*sin(
2*b*x + 2*a))/b
```

Fricas [A] time = 1.64133, size = 120, normalized size = 1.79

$$\frac{15bx+\left(8\cos(bx+a)^5+10\cos(bx+a)^3+15\cos(bx+a)\right)\sin(bx+a)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*b*x + (8*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin
(b*x + a))/b
```

Sympy [A] time = 3.8272, size = 139, normalized size = 2.07

$$\begin{cases}\frac{5x\sin^6(a+bx)}{16}+\frac{15x\sin^4(a+bx)\cos^2(a+bx)}{16}+\frac{15x\sin^2(a+bx)\cos^4(a+bx)}{16}+\frac{5x\cos^6(a+bx)}{16}+\frac{5\sin^5(a+bx)\cos(a+bx)}{16b}+\frac{5\sin^3(a+bx)\cos^3(a+b}{6b}\\ x\cos^6(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**6/16 + 15*x*sin(a + b*x)**4*cos(a + b*x)**2/16
 + 15*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + 5*x*cos(a + b*x)**6/16 + 5*sin
(a + b*x)**5*cos(a + b*x)/(16*b) + 5*sin(a + b*x)**3*cos(a + b*x)**3/(6*b)
+ 11*sin(a + b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*cos(a)**6, True))
```

Giac [A] time = 1.18822, size = 62, normalized size = 0.93

$$\frac{5}{16}\,x + \frac{\sin\left(6\,bx + 6\,a\right)}{192\,b} + \frac{3\,\sin\left(4\,bx + 4\,a\right)}{64\,b} + \frac{15\,\sin\left(2\,bx + 2\,a\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 5/16*x + 1/192*sin(6*b*x + 6*a)/b + 3/64*sin(4*b*x + 4*a)/b + 15/64*sin(2*b
*x + 2*a)/b
```

3.7 $\int \cos^7(a+bx)\,dx$

Optimal. Leaf size=54

$$-\frac{\sin^7(a+bx)}{7b}+\frac{3\sin^5(a+bx)}{5b}-\frac{\sin^3(a+bx)}{b}+\frac{\sin(a+bx)}{b}$$

```
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/b + (3*Sin[a + b*x]^5)/(5*b) - Sin[a + b*x]
^7/(7*b)
```

Rubi [A] time = 0.0163204, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$-\frac{\sin^7(a+bx)}{7b}+\frac{3\sin^5(a+bx)}{5b}-\frac{\sin^3(a+bx)}{b}+\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^7,x]
```

```
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/b + (3*Sin[a + b*x]^5)/(5*b) - Sin[a + b*x]
^7/(7*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{b}+\frac{3\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0132348, size = 54, normalized size = 1.

$$-\frac{\sin^7(a+bx)}{7b}+\frac{3\sin^5(a+bx)}{5b}-\frac{\sin^3(a+bx)}{b}+\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^7,x]
```

```
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/b + (3*Sin[a + b*x]^5)/(5*b) - Sin[a + b*x]
^7/(7*b)
```

Maple [A] time = 0.026, size = 42, normalized size = 0.8

$$\frac{\sin(bx+a)}{7b}\left(\frac{16}{5}+(\cos(bx+a))^6+\frac{6(\cos(bx+a))^4}{5}+\frac{8(\cos(bx+a))^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7,x)
```

```
[Out] 1/7/b*(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A] time = 1.63903, size = 59, normalized size = 1.09

$$-\frac{5 \sin(bx+a)^7 - 21 \sin(bx+a)^5 + 35 \sin(bx+a)^3 - 35 \sin(bx+a)}{35 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/35*(5*sin(b*x + a)^7 - 21*sin(b*x + a)^5 + 35*sin(b*x + a)^3 - 35*sin(b*
x + a))/b
```

Fricas [A] time = 1.68887, size = 112, normalized size = 2.07

$$\frac{\left(5 \cos(bx+a)^6 + 6 \cos(bx+a)^4 + 8 \cos(bx+a)^2 + 16\right) \sin(bx+a)}{35 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x
+ a)/b
```

Sympy [A] time = 6.8842, size = 78, normalized size = 1.44

$$\begin{cases} \frac{16 \sin^7(a+bx)}{35b} + \frac{8 \sin^5(a+bx)\cos^2(a+bx)}{5b} + \frac{2 \sin^3(a+bx)\cos^4(a+bx)}{b} + \frac{\sin(a+bx)\cos^6(a+bx)}{b} & \text{for } b \neq 0 \\ x \cos^7(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7,x)
```

```
[Out] Piecewise((16*sin(a + b*x)**7/(35*b) + 8*sin(a + b*x)**5*cos(a + b*x)**2/(5
*b) + 2*sin(a + b*x)**3*cos(a + b*x)**4/b + sin(a + b*x)*cos(a + b*x)**6/b,
 Ne(b, 0)), (x*cos(a)**7, True))
```

Giac [A] time = 1.51886, size = 59, normalized size = 1.09

$$-\frac{5 \sin(bx+a)^7 - 21 \sin(bx+a)^5 + 35 \sin(bx+a)^3 - 35 \sin(bx+a)}{35 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/35*(5*sin(b*x + a)^7 - 21*sin(b*x + a)^5 + 35*sin(b*x + a)^3 - 35*sin(b*
x + a))/b
```

3.8 $\int \cos^8(a+bx)\,dx$

Optimal. Leaf size=88

$$\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{7\sin(a+bx)\cos^5(a+bx)}{48b}+\frac{35\sin(a+bx)\cos^3(a+bx)}{192b}+\frac{35\sin(a+bx)\cos(a+bx)}{128b}+\frac{35x}{128}$$

```
[Out] (35*x)/128 + (35*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (35*Cos[a + b*x]^3*Si
n[a + b*x])/(192*b) + (7*Cos[a + b*x]^5*Sin[a + b*x])/(48*b) + (Cos[a + b*x
]^7*Sin[a + b*x])/(8*b)
```

Rubi [A] time = 0.0469793, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{7\sin(a+bx)\cos^5(a+bx)}{48b}+\frac{35\sin(a+bx)\cos^3(a+bx)}{192b}+\frac{35\sin(a+bx)\cos(a+bx)}{128b}+\frac{35x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^8,x]
```

```
[Out] (35*x)/128 + (35*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (35*Cos[a + b*x]^3*Si
n[a + b*x])/(192*b) + (7*Cos[a + b*x]^5*Sin[a + b*x])/(48*b) + (Cos[a + b*x
]^7*Sin[a + b*x])/(8*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^8(a+bx)\,dx &= \frac{\cos^7(a+bx)\sin(a+bx)}{8b}+\frac{7}{8}\int \cos^6(a+bx)\,dx\\
&= \frac{7\cos^5(a+bx)\sin(a+bx)}{48b}+\frac{\cos^7(a+bx)\sin(a+bx)}{8b}+\frac{35}{48}\int \cos^4(a+bx)\,dx\\
&= \frac{35\cos^3(a+bx)\sin(a+bx)}{192b}+\frac{7\cos^5(a+bx)\sin(a+bx)}{48b}+\frac{\cos^7(a+bx)\sin(a+bx)}{8b}+\frac{35}{64}\int \cos^2(a+bx)\,dx\\
&= \frac{35\cos(a+bx)\sin(a+bx)}{128b}+\frac{35\cos^3(a+bx)\sin(a+bx)}{192b}+\frac{7\cos^5(a+bx)\sin(a+bx)}{48b}+\frac{\cos^7(a+bx)\sin(a+bx)}{8b}+\frac{35}{128}\int 1\,dx\\
&= \frac{35x}{128}+\frac{35\cos(a+bx)\sin(a+bx)}{128b}+\frac{35\cos^3(a+bx)\sin(a+bx)}{192b}+\frac{7\cos^5(a+bx)\sin(a+bx)}{48b}+\frac{\cos^7(a+bx)\sin(a+bx)}{8b}
\end{aligned}$$

Mathematica [A] time = 0.0545457, size = 55, normalized size = 0.62

$$\frac{672\sin(2(a+bx))+168\sin(4(a+bx))+32\sin(6(a+bx))+3\sin(8(a+bx))+840a+840bx}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^8,x]
```

```
[Out] (840*a + 840*b*x + 672*Sin[2*(a + b*x)] + 168*Sin[4*(a + b*x)] + 32*Sin[6*(
a + b*x)] + 3*Sin[8*(a + b*x)])/(3072*b)
```

Maple [A] time = 0.026, size = 58, normalized size = 0.7

$$\frac{1}{b}\left(\frac{\sin(bx+a)}{8}\left((\cos(bx+a))^7+\frac{7(\cos(bx+a))^5}{6}+\frac{35(\cos(bx+a))^3}{24}+\frac{35\cos(bx+a)}{16}\right)+\frac{35bx}{128}+\frac{35a}{128}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^8,x)
```

```
[Out] 1/b*(1/8*(cos(b*x+a)^7+7/6*cos(b*x+a)^5+35/24*cos(b*x+a)^3+35/16*cos(b*x+a)
)*sin(b*x+a)+35/128*b*x+35/128*a)
```

Maxima [A] time = 1.63483, size = 80, normalized size = 0.91

$$-\frac{128\sin(2bx+2a)^3-840bx-840a-3\sin(8bx+8a)-168\sin(4bx+4a)-768\sin(2bx+2a)}{3072b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/3072*(128*sin(2*b*x + 2*a)^3 - 840*b*x - 840*a - 3*sin(8*b*x + 8*a) - 16
8*sin(4*b*x + 4*a) - 768*sin(2*b*x + 2*a))/b
```

Fricas [A] time = 1.43896, size = 153, normalized size = 1.74

$$\frac{105bx+\left(48\cos(bx+a)^7+56\cos(bx+a)^5+70\cos(bx+a)^3+105\cos(bx+a)\right)\sin(bx+a)}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/384*(105*b*x + (48*cos(b*x + a)^7 + 56*cos(b*x + a)^5 + 70*cos(b*x + a)^3
 + 105*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [A] time = 12.0218, size = 184, normalized size = 2.09

$$\begin{cases}\frac{35x\sin^8(a+bx)}{128}+\frac{35x\sin^6(a+bx)\cos^2(a+bx)}{32}+\frac{105x\sin^4(a+bx)\cos^4(a+bx)}{64}+\frac{35x\sin^2(a+bx)\cos^6(a+bx)}{32}+\frac{35x\cos^8(a+bx)}{128}+\frac{35\sin^7(a+bx)\cos}{128b} \\ x\cos^8(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**8,x)
```

```
[Out] Piecewise((35*x*sin(a + b*x)**8/128 + 35*x*sin(a + b*x)**6*cos(a + b*x)**2/
32 + 105*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 35*x*sin(a + b*x)**2*cos(a
+ b*x)**6/32 + 35*x*cos(a + b*x)**8/128 + 35*sin(a + b*x)**7*cos(a + b*x)/(
128*b) + 385*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) + 511*sin(a + b*x)**3*
cos(a + b*x)**5/(384*b) + 93*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)
), (x*cos(a)**8, True))
```

Giac [A] time = 1.38629, size = 81, normalized size = 0.92

$$\frac{35}{128}\,x+\frac{\sin\left(8\,bx+8\,a\right)}{1024\,b}+\frac{\sin\left(6\,bx+6\,a\right)}{96\,b}+\frac{7\,\sin\left(4\,bx+4\,a\right)}{128\,b}+\frac{7\,\sin\left(2\,bx+2\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 35/128*x + 1/1024*sin(8*b*x + 8*a)/b + 1/96*sin(6*b*x + 6*a)/b + 7/128*sin(
4*b*x + 4*a)/b + 7/32*sin(2*b*x + 2*a)/b
```

3.9 $\int \cos^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{10F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b}+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}+\frac{10\sin(a+bx)\sqrt{\cos(a+bx)}}{21b}$$

```
[Out] (10*EllipticF[(a + b*x)/2, 2])/(21*b) + (10*Sqrt[Cos[a + b*x]]*Sin[a + b*x]
)/(21*b) + (2*Cos[a + b*x]^(5/2)*Sin[a + b*x])/(7*b)
```

Rubi [A] time = 0.0316657, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2635, 2641}

$$\frac{10F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b}+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}+\frac{10\sin(a+bx)\sqrt{\cos(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(7/2),x]
```

```
[Out] (10*EllipticF[(a + b*x)/2, 2])/(21*b) + (10*Sqrt[Cos[a + b*x]]*Sin[a + b*x]
)/(21*b) + (2*Cos[a + b*x]^(5/2)*Sin[a + b*x])/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{7}{2}}(a+bx)\,dx &= \frac{2\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{7b}+\frac{5}{7}\int \cos^{\frac{3}{2}}(a+bx)\,dx\\ &= \frac{10\sqrt{\cos(a+bx)}\sin(a+bx)}{21b}+\frac{2\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{7b}+\frac{5}{21}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\ &= \frac{10F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b}+\frac{10\sqrt{\cos(a+bx)}\sin(a+bx)}{21b}+\frac{2\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.109926, size = 51, normalized size = 0.78

$$\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)+(23\sin(a+bx)+3\sin(3(a+bx)))\sqrt{\cos(a+bx)}}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(7/2),x]
```

```
[Out] (20*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*(23*Sin[a + b*x] + 3*Sin
[3*(a + b*x)]))/(42*b)
```

Maple [B] time = 2.068, size = 199, normalized size = 3.1

$$-\frac{2}{21\,b}\sqrt{\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\left(48\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{9}-120\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{7}+128\,\left(\cos\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(7/2),x)
```

```
[Out] -2/21*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(48*cos(1/2*b
*x+1/2*a)^9-120*cos(1/2*b*x+1/2*a)^7+128*cos(1/2*b*x+1/2*a)^5-72*cos(1/2*b*
x+1/2*a)^3+5*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)
*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+16*cos(1/2*b*x+1/2*a))/(-2*sin(1/2*b
*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1
/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(bx+a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(bx+a\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.10 $\int \cos^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}$$

```
[Out] (6*EllipticE[(a + b*x)/2, 2])/(5*b) + (2*Cos[a + b*x]^(3/2)*Sin[a + b*x])/(
5*b)
```

Rubi [A] time = 0.0181638, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2635, 2639}

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(5/2),x]
```

```
[Out] (6*EllipticE[(a + b*x)/2, 2])/(5*b) + (2*Cos[a + b*x]^(3/2)*Sin[a + b*x])/(
5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{5}{2}}(a+bx)\,dx &= \frac{2\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{5b}+\frac{3}{5}\int \sqrt{\cos(a+bx)}\,dx\\ &= \frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.049645, size = 40, normalized size = 0.95

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(2(a+bx))\sqrt{\cos(a+bx)}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(5/2),x]
```

```
[Out] (6*EllipticE[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*Sin[2*(a + b*x)])/(5*b)
```

Maple [B] time = 1.908, size = 202, normalized size = 4.8

$$-\frac{2}{5b}\sqrt{\left(2\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\left(-8\left(\sin\left(1/2\,bx+a/2\right)\right)^{6}\cos\left(1/2\,bx+a/2\right)+8\cos\left(1/2\,bx+a/2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(5/2),x)
```

```
[Out] -2/5*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(-8*sin(1/2*b*
x+1/2*a)^6*cos(1/2*b*x+1/2*a)+8*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4-3*(
2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(
1/2*b*x+1/2*a),2^(1/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/(-2*sin(
1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*
b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(bx+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos\left(bx+a\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(5/2), x)
```

3.11 $\int \cos^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(
3*b)
```

Rubi [A] time = 0.0182761, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2635, 2641}

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(
3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(a+bx)\,dx &= \frac{2\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\ &= \frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0434417, size = 36, normalized size = 0.86

$$\frac{2\left(F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(a+bx)\sqrt{\cos(a+bx)}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*(EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*Sin[a + b*x]))/(3*b)
```

Maple [B] time = 1.506, size = 179, normalized size = 4.3

$$-\frac{2}{3b}\sqrt{\left(2\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\left(4\cos\left(1/2\,bx+a/2\right)\left(\sin\left(1/2\,bx+a/2\right)\right)^{4}+\sqrt{\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\sqrt{2\,(\mathrm{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*cos(1/2*b*x
+1/2*a)*sin(1/2*b*x+1/2*a)^4+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/
2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))-2*sin(1/2*b*x+1/2*a)^
2*cos(1/2*b*x+1/2*a))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/
sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos\left(bx+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos{(bx + a)}^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2), x)
```

3.12 $\int \sqrt{\cos(a+bx)}\,dx$

Optimal. Leaf size=16

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

[Out] (2*EllipticE[(a + b*x)/2, 2])/b

Rubi [A] time = 0.0086084, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2639}

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[a + b*x]],x]

[Out] (2*EllipticE[(a + b*x)/2, 2])/b

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Mathematica [A] time = 0.0185321, size = 16, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[a + b*x]],x]

[Out] (2*EllipticE[(a + b*x)/2, 2])/b

Maple [B] time = 1.359, size = 133, normalized size = 8.3

$$2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}+1}\mathrm{EllipticE}\left(\cos\left(1\right.\right.}{\sqrt{-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{4}+\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}\sin\left(1/2\,bx+a/2\right)\sqrt{2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/2*
a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),
2^(1/2))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1
/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos (bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\cos (bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos (a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos (bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(b*x + a)), x)
```

3.13 $\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=16

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Rubi [A] time = 0.0091156, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2641}

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Mathematica [A] time = 0.0307141, size = 16, normalized size = 1.

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Maple [C] time = 0.037, size = 18, normalized size = 1.1

$$2\,\frac{\text{InverseJacobiAM}\left(1/2\,bx+a/2,\sqrt{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(1/2),x)
```

```
[Out] 2/b*InverseJacobiAM(1/2*b*x+1/2*a,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{\cos(bx+a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cos(b*x + a)), x)
```

3.14 $\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rubi [A] time = 0.0177816, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2636, 2639}

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\cos(a+bx)}\,dx\\ &= -\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}+\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0525389, size = 38, normalized size = 1.

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Maple [A] time = 1.684, size = 101, normalized size = 2.7

$$-2\,\frac{\sqrt{2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}\mathrm{EllipticE}\left(\cos\left(1/2\,bx+a/2\right),\sqrt{2}\right)-2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}\cos(}{\sin\left(1/2\,bx+a/2\right)\sqrt{2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(3/2),x)
```

```
[Out] -2*((2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE
(cos(1/2*b*x+1/2*a),2^(1/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/sin
(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\cos\left(bx+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(-3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-3/2), x)
```

3.15 $\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)} dx$

Optimal. Leaf size=42

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sin[a + b*x])/(3*b*Cos[a + b*x]^(3
/2))
```

Rubi [A] time = 0.0183912, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2636, 2641}

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-5/2),x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sin[a + b*x])/(3*b*Cos[a + b*x]^(3
/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)} dx &= \frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}+\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}} dx\\ &= \frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0596565, size = 36, normalized size = 0.86

$$\frac{2\left(F\left(\frac{1}{2}(a+bx)\middle|2\right)+\frac{\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-5/2),x]
```

```
[Out] (2*(EllipticF[(a + b*x)/2, 2] + Sin[a + b*x]/Cos[a + b*x]^(3/2)))/(3*b)
```

Maple [B] time = 2.024, size = 213, normalized size = 5.1

$$-\frac{2}{3b}\left(-2\sqrt{(\sin(1/2\,bx+a/2))^{2}}\sqrt{2\,(\sin(1/2\,bx+a/2))^{2}-1}\mathrm{EllipticF}\left(\cos(1/2\,bx+a/2),\sqrt{2}\right)(\sin(1/2\,bx+a/2))^{2}+\sqrt{(\mathrm{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(5/2),x)
```

```
[Out] -2/3*(-2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^
2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1
/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))*((2*cos(1/2*b*x+1/2*a)^2-1)
*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)
^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(3/2)/sin(1/2*b*x+1/2*a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\cos(bx+a)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(-5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-5/2), x)
```

3.16 $\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=65

$$-\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{6\sin(a+bx)}{5b\sqrt{\cos(a+bx)}}$$

```
[Out] (-6*EllipticE[(a + b*x)/2, 2])/(5*b) + (2*Sin[a + b*x])/(5*b*Cos[a + b*x]^(
5/2)) + (6*Sin[a + b*x])/(5*b*Sqrt[Cos[a + b*x]])
```

Rubi [A] time = 0.0292644, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2636, 2639}

$$-\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{6\sin(a+bx)}{5b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-7/2),x]
```

```
[Out] (-6*EllipticE[(a + b*x)/2, 2])/(5*b) + (2*Sin[a + b*x])/(5*b*Cos[a + b*x]^(
5/2)) + (6*Sin[a + b*x])/(5*b*Sqrt[Cos[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx\\
&= \frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{6\sin(a+bx)}{5b\sqrt{\cos(a+bx)}}-\frac{3}{5}\int \sqrt{\cos(a+bx)}\,dx\\
&= -\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{6\sin(a+bx)}{5b\sqrt{\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.0932566, size = 59, normalized size = 0.91

$$\frac{3\sin(2(a+bx))+2\tan(a+bx)-6\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-7/2),x]
```

```
[Out] (-6*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] + 3*Sin[2*(a + b*x)] + 2*T
an[a + b*x])/(5*b*Cos[a + b*x]^(3/2))
```

Maple [B] time = 3.014, size = 358, normalized size = 5.5

$$\frac{2}{5b}\sqrt{-\left(-2\left(\cos\left(1/2\,bx+a/2\right)\right)^2+1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(12\sqrt{2\left(\sin\left(1/2\,bx+a/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,bx+a/2\right)\right)^2}\mathrm{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(7/2),x)
```

```
[Out] 2/5*(-(-2*cos(1/2*b*x+1/2*a)^2+1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(8*sin(1/2*b*
x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/sin(1/2*b*x+1/
2*a)^3*(12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*El
lipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4-24*sin(1/2*b*x+1/2
*a)^6*cos(1/2*b*x+1/2*a)-12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1
/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+2
4*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+3*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2
)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-8*sin(
1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1
/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\cos\left(bx+a\right)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(-7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-7/2), x)
```

3.17 $\int (c\cos(a+bx))^{7/2}\,dx$

Optimal. Leaf size=98

$$\frac{10c^3\sin(a+bx)\sqrt{c\cos(a+bx)}}{21b}+\frac{10c^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)(c\cos(a+bx))^{5/2}}{7b}$$

```
[Out] (10*c^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*Sqrt[c*Cos[a +
b*x]]) + (10*c^3*Sqrt[c*Cos[a + b*x]]*Sin[a + b*x])/(21*b) + (2*c*(c*Cos[a
+ b*x])^(5/2)*Sin[a + b*x])/(7*b)
```

Rubi [A] time = 0.0586644, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2642, 2641}

$$\frac{10c^3\sin(a+bx)\sqrt{c\cos(a+bx)}}{21b}+\frac{10c^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)(c\cos(a+bx))^{5/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(7/2),x]
```

```
[Out] (10*c^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*Sqrt[c*Cos[a +
b*x]]) + (10*c^3*Sqrt[c*Cos[a + b*x]]*Sin[a + b*x])/(21*b) + (2*c*(c*Cos[a
+ b*x])^(5/2)*Sin[a + b*x])/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c\cos(a+bx))^{7/2}\,dx = \frac{2c(c\cos(a+bx))^{5/2}\sin(a+bx)}{7b} + \frac{1}{7}\left(5c^2\right)\int (c\cos(a+bx))^{3/2}\,dx$$
$$= \frac{10c^3\sqrt{c\cos(a+bx)}\sin(a+bx)}{21b} + \frac{2c(c\cos(a+bx))^{5/2}\sin(a+bx)}{7b} + \frac{1}{21}\left(5c^4\right)\int \frac{1}{\sqrt{c\cos(a+bx)}}$$
$$= \frac{10c^3\sqrt{c\cos(a+bx)}\sin(a+bx)}{21b} + \frac{2c(c\cos(a+bx))^{5/2}\sin(a+bx)}{7b} + \frac{\left(5c^4\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(}}}{21\sqrt{c\cos(a+bx)}}$$
$$= \frac{10c^4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b\sqrt{c\cos(a+bx)}} + \frac{10c^3\sqrt{c\cos(a+bx)}\sin(a+bx)}{21b} + \frac{2c(c\cos(a+bx))^{5/2}\sin(}{7b}$$

Mathematica [A] time = 0.0920171, size = 76, normalized size = 0.78

$$\frac{c^3\sqrt{c\cos(a+bx)}\left(20F\left(\frac{1}{2}(a+bx)\middle|2\right)+(23\sin(a+bx)+3\sin(3(a+bx)))\sqrt{\cos(a+bx)}\right)}{42b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(7/2),x]
```

```
[Out] (c^3*Sqrt[c*Cos[a + b*x]]*(20*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]
]*(23*Sin[a + b*x] + 3*Sin[3*(a + b*x)])))/(42*b*Sqrt[Cos[a + b*x]])
```

Maple [A] time = 2.047, size = 210, normalized size = 2.1

$$-\frac{2\,c^4}{21\,b}\sqrt{c\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(48\,\left(\cos\left(1/2\,bx+a/2\right)\right)^9-120\,\left(\cos\left(1/2\,bx+a/2\right)\right)^7+128\,\left(\cos\left(1/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(7/2),x)
```

```
[Out] -2/21*(c*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c^4*(48*cos
(1/2*b*x+1/2*a)^9-120*cos(1/2*b*x+1/2*a)^7+128*cos(1/2*b*x+1/2*a)^5-72*cos(
1/2*b*x+1/2*a)^3+5*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)
^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+16*cos(1/2*b*x+1/2*a))/(-c*(2*
sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(c*(2*
cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c\cos\left(bx+a\right)}c^3\cos\left(bx+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*cos(b*x + a))*c^3*cos(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.18 $\int (c\cos(a+bx))^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{6c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}+\frac{2c\sin(a+bx)(c\cos(a+bx))^{3/2}}{5b}$$

```
[Out] (6*c^2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*
x]]) + (2*c*(c*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b)
```

Rubi [A] time = 0.0364114, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2640, 2639}

$$\frac{6c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}+\frac{2c\sin(a+bx)(c\cos(a+bx))^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(5/2),x]
```

```
[Out] (6*c^2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*
x]]) + (2*c*(c*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c\cos(a+bx))^{5/2}\,dx &= \frac{2c(c\cos(a+bx))^{3/2}\sin(a+bx)}{5b}+\frac{1}{5}\left(3c^2\right)\int \sqrt{c\cos(a+bx)}\,dx\\ &= \frac{2c(c\cos(a+bx))^{3/2}\sin(a+bx)}{5b}+\frac{\left(3c^2\sqrt{c\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5\sqrt{\cos(a+bx)}}\\ &= \frac{6c^2\sqrt{c\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}}+\frac{2c(c\cos(a+bx))^{3/2}\sin(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0844326, size = 62, normalized size = 0.89

$$\frac{(c\cos(a+bx))^{5/2}\left(6E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(2(a+bx))\sqrt{\cos(a+bx)}\right)}{5b\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((c*Cos[a + b*x])^(5/2)*(6*EllipticE[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*S
in[2*(a + b*x)]))/(5*b*Cos[a + b*x]^(5/2))
```

Maple [B] time = 1.855, size = 213, normalized size = 3.

$$-\frac{2c^3}{5b}\sqrt{c\left(2\left(\cos(1/2\,bx+a/2)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(-8\left(\sin(1/2\,bx+a/2)\right)^6\cos(1/2\,bx+a/2)+8\cos(1/2\,bx+a/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(5/2),x)
```

```
[Out] -2/5*(c*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c^3*(-8*sin(
1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+8*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)
^4-3*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*Elliptic
E(cos(1/2*b*x+1/2*a),2^(1/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/(-
c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(
c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c\cos(bx+a)}c^2\cos(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*cos(b*x + a))*c^2*cos(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx + a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(5/2), x)
```

3.19 $\int (c\cos(a+bx))^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{2c^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}$$

```
[Out] (2*c^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[c*Cos[a + b*
x]]) + (2*c*Sqrt[c*Cos[a + b*x]]*Sin[a + b*x])/(3*b)
```

Rubi [A] time = 0.0365162, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2642, 2641}

$$\frac{2c^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*c^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[c*Cos[a + b*
x]]) + (2*c*Sqrt[c*Cos[a + b*x]]*Sin[a + b*x])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c\cos(a+bx))^{3/2}\,dx&=\frac{2c\sqrt{c\cos(a+bx)}\sin(a+bx)}{3b}+\frac{1}{3}c^2\int\frac{1}{\sqrt{c\cos(a+bx)}}\,dx\\&=\frac{2c\sqrt{c\cos(a+bx)}\sin(a+bx)}{3b}+\frac{\left(c^2\sqrt{\cos(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3\sqrt{c\cos(a+bx)}}\\&=\frac{2c^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{c\cos(a+bx)}}+\frac{2c\sqrt{c\cos(a+bx)}\sin(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0462831, size = 58, normalized size = 0.83

$$\frac{2(c\cos(a+bx))^{3/2}\left(F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(a+bx)\sqrt{\cos(a+bx)}\right)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*(c*Cos[a + b*x])^(3/2)*(EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*S
in[a + b*x]))/(3*b*Cos[a + b*x]^(3/2))
```

Maple [B] time = 1.884, size = 190, normalized size = 2.7

$$-\frac{2c^2}{3b}\sqrt{c\left(2\left(\cos(1/2bx+a/2)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(4\cos(1/2bx+a/2)\left(\sin(1/2bx+a/2)\right)^4+\sqrt{\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(3/2),x)
```

```
[Out] -2/3*(c*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c^2*(4*cos(1
/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*
b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))-2*sin(1/2*b*x+1
/2*a)^2*cos(1/2*b*x+1/2*a))/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^
2))^(1/2)/sin(1/2*b*x+1/2*a)/(c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(c\cos(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c\cos(bx+a)}c\cos(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*cos(b*x + a))*c*cos(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(3/2), x)
```

3.20 $\int \sqrt{c\cos(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

```
[Out] (2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Rubi [A] time = 0.0215331, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2640, 2639}

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Cos[a + b*x]],x]
```

```
[Out] (2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c\cos(a+bx)}\,dx &= \frac{\sqrt{c\cos(a+bx)}\int \sqrt{\cos(a+bx)}\,dx}{\sqrt{\cos(a+bx)}}\\ &= \frac{2\sqrt{c\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0189257, size = 38, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Cos[a + b*x]],x]
```

```
[Out] (2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Maple [B] time = 1.457, size = 142, normalized size = 3.7

$$2\,\frac{\sqrt{c\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}c\sqrt{\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}+1}\mathrm{EllipticE}\left(\right.}{\sqrt{-c\left(2\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{4}-\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}\right)}\sin\left(1/2\,bx+a/2\right)\sqrt{c\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(1/2),x)
```

```
[Out] 2*(c*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c*(sin(1/2*b*x+
1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2
*a),2^(1/2))/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1
/2*b*x+1/2*a)/(c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\cos\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\cos\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \cos(bx + a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*cos(b*x + a)), x)
```

3.21 $\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{c\cos(a+bx)}}$$

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[c*Cos[a + b*x]])
```

Rubi [A] time = 0.0226469, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2642, 2641}

$$\frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Cos[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[c*Cos[a + b*x]])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx &= \frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{\sqrt{c\cos(a+bx)}}\\ &= \frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{c\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.02488, size = 38, normalized size = 1.

$$\frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Cos[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[c*Cos[a + b*x]])
```

Maple [C] time = 0.204, size = 54, normalized size = 1.4

$$2\,{\frac {\sqrt {2\, \left(\cos \left(1/2\,bx+a/2 \right) \right) ^{2}-1}{\it InverseJacobiAM} \left(1/2\,bx+a/2,\sqrt {2} \right) }{b\sqrt {c \left(2\, \left(\cos \left(1/2\,bx+a/2 \right) \right) ^{2}-1 \right) }}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(1/2),x)
```

```
[Out] 2/b/(c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)*(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)*I
nverseJacobiAM(1/2*b*x+1/2*a,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c \cos \left(bx + a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{c \cos \left(bx + a\right)}}{c \cos \left(bx + a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*cos(b*x + a))/(c*cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c \cos{\left (a + bx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*cos(b*x + a)), x)
```

3.22 $\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{bc^2\sqrt{\cos(a+bx)}}$$

```
[Out] (-2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*c^2*Sqrt[Cos[a + b*x
]]) + (2*Sin[a + b*x])/(b*c*Sqrt[c*Cos[a + b*x]])
```

Rubi [A] time = 0.0360195, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2640, 2639}

$$\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{bc^2\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*c^2*Sqrt[Cos[a + b*x
]]) + (2*Sin[a + b*x])/(b*c*Sqrt[c*Cos[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx &= \frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{\int \sqrt{c\cos(a+bx)}\,dx}{c^2}\\ &= \frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{\sqrt{c\cos(a+bx)}\int \sqrt{\cos(a+bx)}\,dx}{c^2\sqrt{\cos(a+bx)}}\\ &= -\frac{2\sqrt{c\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bc^2\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0303218, size = 50, normalized size = 0.74

$$\frac{2\left(\sin(a+bx)-\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{bc\sqrt{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-3/2),x]
```

```
[Out] (2*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]) + Sin[a + b*x]))/(b*c*S
qrt[c*Cos[a + b*x]])
```

Maple [A] time = 2.045, size = 168, normalized size = 2.5

$$-2\,\frac{\sqrt{-2\,c\,(\sin(1/2\,bx+a/2))^4+c\,(\sin(1/2\,bx+a/2))^2}\left(\sqrt{2\,(\sin(1/2\,bx+a/2))^2-1}\sqrt{(\sin(1/2\,bx+a/2))^2}\mathrm{EllipticE}\,(\mathrm{c}\right.}{c\sqrt{-c\left(2\,(\sin(1/2\,bx+a/2))^4-(\sin(1/2\,bx+a/2))^2\right)}\sin(1/2\,bx+a/2)\sqrt{c\,(2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(3/2),x)
```

```
[Out] -2/c*(-2*c*sin(1/2*b*x+1/2*a)^4+c*sin(1/2*b*x+1/2*a)^2)^(1/2)*((2*sin(1/2*b
*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2
*a),2^(1/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/(-c*(2*sin(1/2*b*x+
1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(c*(2*cos(1/2*b*x+
1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c\cos(bx+a)}}{c^2\cos(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*cos(b*x + a))/(c^2*cos(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(-3/2), x)
```

3.23 $\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bc^2\sqrt{c\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*c^2*Sqrt[c*Cos[a + b*
x]]) + (2*Sin[a + b*x])/(3*b*c*(c*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0353468, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2642, 2641}

$$\frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bc^2\sqrt{c\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-5/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*c^2*Sqrt[c*Cos[a + b*
x]]) + (2*Sin[a + b*x])/(3*b*c*(c*Cos[a + b*x])^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx &= \frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}+\frac{\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx}{3c^2}\\ &= \frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}+\frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3c^2\sqrt{c\cos(a+bx)}}\\ &= \frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bc^2\sqrt{c\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0591189, size = 51, normalized size = 0.71

$$\frac{2\left(\tan(a+bx)+\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{3bc^2\sqrt{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-5/2),x]
```

```
[Out] (2*(Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Tan[a + b*x]))/(3*b*c^2*
Sqrt[c*Cos[a + b*x]])
```

Maple [B] time = 1.763, size = 241, normalized size = 3.4

$$-\frac{2}{3c^2b}\left(-2\sqrt{(\sin(1/2bx+a/2))^2}\sqrt{2(\sin(1/2bx+a/2))^2-1}\text{EllipticF}\left(\cos(1/2bx+a/2),\sqrt{2}\right)(\sin(1/2bx+a/2))^2+\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(5/2),x)
```

```
[Out] -2/3*(-2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^
2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1
/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/c^2*(c*(2*cos(1/2*b*x+1/2*a
)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+
1/2*a)^2))^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)/sin(1/2*b*x+1/2*a)/(c*(2*cos(1/
2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\cos(bx+a)}}{c^3\cos(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*cos(b*x + a))/(c^3*cos(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(-5/2), x)
```

3.24 $\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}}-\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{5bc^4\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}$$

```
[Out] (-6*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*c^4*Sqrt[Cos[a + b
*x]]) + (2*Sin[a + b*x])/(5*b*c*(c*Cos[a + b*x])^(5/2)) + (6*Sin[a + b*x])/
(5*b*c^3*Sqrt[c*Cos[a + b*x]])
```

Rubi [A] time = 0.0556972, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2640, 2639}

$$\frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}}-\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{5bc^4\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*c^4*Sqrt[Cos[a + b
*x]]) + (2*Sin[a + b*x])/(5*b*c*(c*Cos[a + b*x])^(5/2)) + (6*Sin[a + b*x])/
(5*b*c^3*Sqrt[c*Cos[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx = \frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}} + \frac{3\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx}{5c^2}$$
$$= \frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}} + \frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}} - \frac{3\int \sqrt{c\cos(a+bx)}\,dx}{5c^4}$$
$$= \frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}} + \frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}} - \frac{\left(3\sqrt{c\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5c^4\sqrt{\cos(a+bx)}}$$
$$= -\frac{6\sqrt{c\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^4\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}} + \frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}}$$

Mathematica [A] time = 0.0947585, size = 68, normalized size = 0.68

$$\frac{6\sin(a+bx) - 6\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right) + 2\tan(a+bx)\sec(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + 6*Sin[a + b*x] + 2*Sec[a
 + b*x]*Tan[a + b*x])/(5*b*c^3*Sqrt[c*Cos[a + b*x]])
```

Maple [B] time = 3.175, size = 366, normalized size = 3.7

$$\frac{2}{5\,c^4 b}\sqrt{c\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(12\,\sqrt{2\,(\sin(1/2\,bx+a/2))^2-1}\sqrt{(\sin(1/2\,bx+a/2))^2}\mathrm{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(7/2),x)
```

```
[Out] 2/5*(c*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/c^4/sin(1/2*b
*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1
/2*a)^2-1)*(12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2
)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4-24*sin(1/2*b*x
+1/2*a)^6*cos(1/2*b*x+1/2*a)-12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b
*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)
^2+24*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+3*(2*sin(1/2*b*x+1/2*a)^2-1)^
(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-8*
sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))*(-2*c*sin(1/2*b*x+1/2*a)^4+c*sin(1
/2*b*x+1/2*a)^2)^(1/2)/(c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\cos\left(bx+a\right)}}{c^4\cos\left(bx+a\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*cos(b*x + a))/(c^4*cos(b*x + a)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(-7/2), x)
```

3.25 $\int \cos^{\frac{4}{3}}(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{3\sin(a+bx)\cos^{\frac{7}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-3*Cos[a + b*x]^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Si
n[a + b*x])/(7*b*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0125782, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$-\frac{3\sin(a+bx)\cos^{\frac{7}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(4/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Si
n[a + b*x])/(7*b*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{4}{3}}(a+bx)\,dx=-\frac{3\cos^{\frac{7}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)\sin(a+bx)}{7b\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0584261, size = 53, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(a+bx)}\cos^{\frac{7}{3}}(a+bx)\csc(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(4/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(7/3)*Csc[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a
 + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(7*b)
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int (\cos (bx + a))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(4/3),x)
```

```
[Out] int(cos(b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos (bx + a)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos (bx + a)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(4/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos (bx + a)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3),x, algorithm="giac")

[Out] integrate(cos(b*x + a)^(4/3), x)
```

3.26 $\int \cos^{\frac{2}{3}}(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{3\sin(a+bx)\cos^{\frac{5}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-3*Cos[a + b*x]^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Si
n[a + b*x])/(5*b*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0120525, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$-\frac{3\sin(a+bx)\cos^{\frac{5}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(2/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Si
n[a + b*x])/(5*b*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{2}{3}}(a+bx)\,dx = -\frac{3\cos^{\frac{5}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)\sin(a+bx)}{5b\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0334061, size = 53, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(a+bx)}\cos^{\frac{5}{3}}(a+bx)\csc(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(2/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(5/3)*Csc[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a
 + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(5*b)
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int (\cos (bx + a))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(2/3),x)
```

```
[Out] int(cos(b*x+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos (bx + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos (bx + a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos (bx + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(2/3), x)
```

3.27 $\int \sqrt[3]{\cos(a+bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{3\sin(a+bx)\cos^{\frac{4}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-3*Cos[a + b*x]^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin
[a + b*x])/(4*b*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0117492, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$-\frac{3\sin(a+bx)\cos^{\frac{4}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(1/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin
[a + b*x])/(4*b*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{\cos(a+bx)}\,dx = -\frac{3\cos^{\frac{4}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)\sin(a+bx)}{4b\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0285543, size = 53, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(a+bx)}\cos^{\frac{4}{3}}(a+bx)\csc(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(1/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(4/3)*Csc[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a
+ b*x]^2]*Sqrt[Sin[a + b*x]^2])/(4*b)
```

Maple [F] time = 0.157, size = 0, normalized size = 0.

$$\int \sqrt[3]{\cos(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/3),x)
```

```
[Out] int(cos(b*x+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos(bx+a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{\cos(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/3),x)
```

```
[Out] Integral(cos(a + b*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(1/3), x)
```

3.28 $\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx$

Optimal. Leaf size=53

$$-\frac{3\sin(a+bx)\cos^{\frac{2}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-3*Cos[a + b*x]^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin
[a + b*x])/(2*b*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.012273, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$-\frac{3\sin(a+bx)\cos^{\frac{2}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-1/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin
[a + b*x])/(2*b*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx = -\frac{3\cos^{\frac{2}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)\sin(a+bx)}{2b\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0253434, size = 53, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(a+bx)}\cos^{\frac{2}{3}}(a+bx)\csc(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-1/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(2/3)*Csc[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a
+ b*x]^2]*Sqrt[Sin[a + b*x]^2])/(2*b)
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(1/3),x)
```

```
[Out] int(1/cos(b*x+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\cos(bx+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(-1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(1/3),x)
```

```
[Out] Integral(cos(a + b*x)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-1/3), x)
```

3.29 $\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=51

$$-\frac{3\sin(a+bx)\sqrt[3]{\cos(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-3*Cos[a + b*x]^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin
[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0118621, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$-\frac{3\sin(a+bx)\sqrt[3]{\cos(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-2/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin
[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cos(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0247452, size = 51, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(a+bx)}\sqrt[3]{\cos(a+bx)}\csc(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-2/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(1/3)*Csc[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a
+ b*x]^2]*Sqrt[Sin[a + b*x]^2])/b
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int (\cos (bx+a))^{-\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(2/3),x)
```

```
[Out] int(1/cos(b*x+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos (bx+a)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\cos (bx+a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(-2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(2/3),x)
```

```
[Out] Integral(cos(a + b*x)**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos (bx+a)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-2/3), x)
```

3.30 $\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx$

Optimal. Leaf size=51

$$\frac{3\sin(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{\cos(a+bx)}}$$

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*Cos[a
 + b*x]^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0119056, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$\frac{3\sin(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*Cos[a
 + b*x]^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt[3]{\cos(a+bx)}\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0263183, size = 51, normalized size = 1.

$$\frac{3\sqrt{\sin^2(a+bx)}\csc(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{b\sqrt[3]{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-4/3),x]
```

```
[Out] (3*Csc[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sqrt[Sin[
a + b*x]^2])/(b*Cos[a + b*x]^(1/3))
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int (\cos (bx+a))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(4/3),x)
```

```
[Out] int(1/cos(b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos (bx+a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\cos (bx+a)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(-4/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos (bx+a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-4/3), x)
```

3.31 $\int (c\cos(a+bx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7bc\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-3*(c*Cos[a + b*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2
]*Sin[a + b*x])/(7*b*c*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0142855, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7bc\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(4/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2
]*Sin[a + b*x])/(7*b*c*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(a+bx))^{4/3}\,dx = -\frac{3(c\cos(a+bx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)\sin(a+bx)}{7bc\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0487298, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\sin^2(a+bx)}\cot(a+bx)(c\cos(a+bx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(4/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(4/3)*Cot[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6, C
os[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(7*b)
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(4/3),x)
```

```
[Out] int((c*cos(b*x+a))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c\cos(bx+a))^{\frac{1}{3}}c\cos(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(1/3)*c*cos(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(4/3), x)
```

3.32 $\int (c\cos(a+bx))^{2/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5bc\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-3*(c*Cos[a + b*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2
]*Sin[a + b*x])/(5*b*c*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.01522, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5bc\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(2/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2
]*Sin[a + b*x])/(5*b*c*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(a+bx))^{2/3}\,dx=-\frac{3(c\cos(a+bx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)\sin(a+bx)}{5bc\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0381831, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\sin^2(a+bx)}\cot(a+bx)(c\cos(a+bx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(2/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(2/3)*Cot[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6, C
os[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(5*b)
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(2/3),x)
```

```
[Out] int((c*cos(b*x+a))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((c\cos(bx+a))^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(2/3),x, algorithm="giac")

[Out] integrate((c*cos(b*x + a))^(2/3), x)
```

3.33 $\int \sqrt[3]{c\cos(a+bx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4bc\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-3*(c*Cos[a + b*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]
*Sin[a + b*x])/(4*b*c*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0154246, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4bc\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(1/3),x]
```


```
[Out] (-3*(c*Cos[a + b*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]
*Sin[a + b*x])/(4*b*c*Sqrt[Sin[a + b*x]^2])
```


Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{c\cos(a+bx)}\,dx=-\frac{3(c\cos(a+bx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)\sin(a+bx)}{4bc\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0367156, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\sin^2(a+bx)}\cot(a+bx)\sqrt[3]{c\cos(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(1/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(1/3)*Cot[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Co
s[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(4*b)
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int \sqrt[3]{c\cos(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(1/3),x)
```

```
[Out] int((c*cos(b*x+a))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((c\cos(bx+a))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{c\cos(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*cos(a + b*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/3),x, algorithm="giac")

[Out] integrate((c*cos(b*x + a))^(1/3), x)
```

3.34 $\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2bc\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-3*(c*Cos[a + b*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]
*Sin[a + b*x])/(2*b*c*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0153915, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2bc\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-1/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]
*Sin[a + b*x])/(2*b*c*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx = -\frac{3(c\cos(a+bx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)\sin(a+bx)}{2bc\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0413037, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\sin^2(a+bx)}\cot(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2b\sqrt[3]{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-1/3),x]
```

```
[Out] (-3*Cot[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sqrt[Sin[
a + b*x]^2])/(2*b*(c*Cos[a + b*x])^(1/3))
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(1/3),x)
```

```
[Out] int(1/(c*cos(b*x+a))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\cos(bx+a)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c\cos(bx+a)\right)^{\frac{2}{3}}}{c\cos(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(2/3)/(c*cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*cos(a + b*x))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\cos(bx+a)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(-1/3), x)
```

3.35 $\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sin(a+bx)\sqrt[3]{c\cos(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{bc\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-3*(c*Cos[a + b*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]
*Sin[a + b*x])/(b*c*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0148829, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\sin(a+bx)\sqrt[3]{c\cos(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{bc\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-2/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]
*Sin[a + b*x])/(b*c*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx = -\frac{3\sqrt[3]{c\cos(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)\sin(a+bx)}{bc\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0437079, size = 53, normalized size = 0.95

$$-\frac{3\sqrt{\sin^2(a+bx)}\cot(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{b(c\cos(a+bx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-2/3),x]
```

```
[Out] (-3*Cot[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sqrt[Sin[
a + b*x]^2])/(b*(c*Cos[a + b*x])^(2/3))
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(2/3),x)
```

```
[Out] int(1/(c*cos(b*x+a))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cos(bx+a))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(c\cos(bx+a))^{\frac{1}{3}}}{c\cos(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(1/3)/(c*cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cos(a+bx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*cos(a + b*x))**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cos(bx+a))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(-2/3), x)
```

3.36 $\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$\frac{3\sin(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{bc\sqrt{\sin^2(a+bx)}\sqrt[3]{c\cos(a+bx)}}$$

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*c*(c*
Cos[a + b*x])^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0250676, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\sin(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{bc\sqrt{\sin^2(a+bx)}\sqrt[3]{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*c*(c*
Cos[a + b*x])^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx=\frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)\sin(a+bx)}{bc\sqrt[3]{c\cos(a+bx)}\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0411307, size = 53, normalized size = 0.95

$$\frac{3\sqrt{\sin^2(a+bx)}\cot(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{b(c\cos(a+bx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-4/3),x]
```

```
[Out] (3*Cot[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sqrt[Sin[
a + b*x]^2])/(b*(c*Cos[a + b*x])^(4/3))
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(4/3),x)
```

```
[Out] int(1/(c*cos(b*x+a))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cos(bx+a))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(c\cos(bx+a))^{\frac{2}{3}}}{c^2\cos(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(2/3)/(c^2*cos(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cos(bx+a))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(-4/3), x)
```

3.37 $\int \cos^n(a+bx)\,dx$

Optimal. Leaf size=64

$$-\frac{\sin(a+bx)\cos^{n+1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -((Cos[a + b*x]^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[a
+ b*x]^2]*Sin[a + b*x])/(b*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rubi [A] time = 0.0170212, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2643}

$$-\frac{\sin(a+bx)\cos^{n+1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^n,x]
```

```
[Out] -((Cos[a + b*x]^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[a
+ b*x]^2]*Sin[a + b*x])/(b*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^n(a+bx)\,dx = -\frac{\cos^{1+n}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin(a+bx)}{b(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0456161, size = 64, normalized size = 1.

$$-\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)\cos^{n+1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^n,x]
```

```
[Out] -((Cos[a + b*x]^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 +
 n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(1 + n)))
```

Maple [F] time = 0.368, size = 0, normalized size = 0.

$$\int (\cos (bx + a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^n,x)
```

```
[Out] int(cos(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos (bx + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos (bx + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^n (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**n,x)
```

```
[Out] Integral(cos(a + b*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos (bx + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^n, x)
```

3.38 $\int (c\cos(a+bx))^n\,dx$

Optimal. Leaf size=69

$$-\frac{\sin(a+bx)(c\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bc(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(((c*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*Sin[a + b*x])/(b*c*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rubi [A] time = 0.0203006, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$-\frac{\sin(a+bx)(c\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bc(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^n,x]
```

```
[Out] -(((c*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*Sin[a + b*x])/(b*c*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(a+bx))^n\,dx = -\frac{(c\cos(a+bx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin(a+bx)}{bc(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.0437238, size = 64, normalized size = 0.93

$$-\frac{\sqrt{\sin^2(a+bx)}\cot(a+bx)(c\cos(a+bx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^n,x]
```

```
[Out] -(((c*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(1 + n)))
```

Maple [F] time = 0.447, size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^n,x)
```

```
[Out] int((c*cos(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c\cos(bx+a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(a+bx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**n,x)
```

```
[Out] Integral((c*cos(a + b*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cos(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^n, x)
```

3.39 $\int \left(a\cos^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2\tan(x)\sqrt{a\cos^2(x)}+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{15}a\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

```
[Out] (8*a^2*Sqrt[a*Cos[x]^2]*Tan[x])/15 + (4*a*(a*Cos[x]^2)^(3/2)*Tan[x])/15 + (
(a*Cos[x]^2)^(5/2)*Tan[x])/5
```

Rubi [A] time = 0.0387325, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3203, 3207, 2637}

$$\frac{8}{15}a^2\tan(x)\sqrt{a\cos^2(x)}+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{15}a\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[a*Cos[x]^2]*Tan[x])/15 + (4*a*(a*Cos[x]^2)^(3/2)*Tan[x])/15 + (
(a*Cos[x]^2)^(5/2)*Tan[x])/5
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a\cos^2(x)\right)^{5/2}\,dx &= \frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{5}(4a)\int \left(a\cos^2(x)\right)^{3/2}\,dx\\
&= \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{15}\left(8a^2\right)\int \sqrt{a\cos^2(x)}\,dx\\
&= \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{15}\left(8a^2\sqrt{a\cos^2(x)}\sec(x)\right)\int \cos(x)\,dx\\
&= \frac{8}{15}a^2\sqrt{a\cos^2(x)}\tan(x)+\frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.019473, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2(150\sin(x)+25\sin(3x)+3\sin(5x))\sec(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^2)^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*Cos[x]^2]*Sec[x]*(150*Sin[x] + 25*Sin[3*x] + 3*Sin[5*x]))/240
```

Maple [A] time = 0.628, size = 32, normalized size = 0.6

$$\frac{a^3\cos(x)\sin(x)\left(3\,(\cos(x))^4+4\,(\cos(x))^2+8\right)}{15}\frac{1}{\sqrt{a(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^2)^(5/2),x)
```

```
[Out] 1/15*a^3*cos(x)*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/(a*cos(x)^2)^(1/2)
```

Maxima [A] time = 2.34722, size = 42, normalized size = 0.79

$$\frac{1}{240}\left(3\,a^2\sin(5\,x)+25\,a^2\sin(3\,x)+150\,a^2\sin(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*a^2*sin(5*x) + 25*a^2*sin(3*x) + 150*a^2*sin(x))*sqrt(a)
```

Fricas [A] time = 1.12342, size = 107, normalized size = 2.02

$$\frac{\left(3\,a^2\cos(x)^4+4\,a^2\cos(x)^2+8\,a^2\right)\sqrt{a\cos(x)^2}\sin(x)}{15\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^2*cos(x)^4 + 4*a^2*cos(x)^2 + 8*a^2)*sqrt(a*cos(x)^2)*sin(x)/cos(
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40348, size = 46, normalized size = 0.87

$$\frac{1}{15}\left(3\,a^2\sin\left(x\right)^5-10\,a^2\sin\left(x\right)^3+15\,a^2\sin\left(x\right)\right)\sqrt{a}\mathrm{sgn}\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*a^2*sin(x)^5 - 10*a^2*sin(x)^3 + 15*a^2*sin(x))*sqrt(a)*sgn(cos(x))
```

3.40 $\int \left(a \cos^2(x)\right)^{3/2} dx$

Optimal. Leaf size=34

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

```
[Out] (2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3
```

Rubi [A] time = 0.0268519, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3203, 3207, 2637}

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^2)^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] && !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a\cos^2(x)\right)^{3/2} dx &= \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{3}(2a)\int \sqrt{a\cos^2(x)}\,dx\\ &= \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{3}\left(2a\sqrt{a\cos^2(x)}\sec(x)\right)\int \cos(x)\,dx\\ &= \frac{2}{3}a\sqrt{a\cos^2(x)}\tan(x)+\frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0094787, size = 26, normalized size = 0.76

$$\frac{1}{12}a(9\sin(x)+\sin(3x))\sec(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^2)^(3/2),x]
```

```
[Out] (a*Sqrt[a*Cos[x]^2]*Sec[x]*(9*Sin[x] + Sin[3*x]))/12
```

Maple [A] time = 0.605, size = 24, normalized size = 0.7

$$\frac{a^2\cos(x)\sin(x)\left((\cos(x))^2+2\right)}{3}\frac{1}{\sqrt{a(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^2)^(3/2),x)
```

```
[Out] 1/3*a^2*cos(x)*sin(x)*(cos(x)^2+2)/(a*cos(x)^2)^(1/2)
```

Maxima [A] time = 2.55931, size = 23, normalized size = 0.68

$$\frac{1}{12}\,(a\sin(3x)+9\,a\sin(x))\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(a*sin(3*x) + 9*a*sin(x))*sqrt(a)
```

Fricas [A] time = 1.04613, size = 74, normalized size = 2.18

$$\frac{\left(a\cos(x)^2+2\,a\right)\sqrt{a\cos(x)^2}\sin(x)}{3\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(x)^2 + 2*a)*sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4166, size = 23, normalized size = 0.68

$$-\frac{1}{3}\left(\sin\left(x\right)^{3}-3\,\sin\left(x\right)\right)a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))*a^(3/2)*sgn(cos(x))
```

3.41 $\int \sqrt{a\cos^2(x)}\,dx$

Optimal. Leaf size=13

$$\tan(x)\sqrt{a\cos^2(x)}$$

```
[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

Rubi [A] time = 0.0112618, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 2637}

$$\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[x]^2],x]
```

```
[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\cos^2(x)}\,dx &= \left(\sqrt{a\cos^2(x)}\sec(x)\right)\int \cos(x)\,dx\\ &= \sqrt{a\cos^2(x)}\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0041028, size = 13, normalized size = 1.

$$\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[x]^2],x]
```

```
[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

Maple [A] time = 0.337, size = 15, normalized size = 1.2

$$a\cos(x)\sin(x)\frac{1}{\sqrt{a(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^2)^(1/2),x)
```

```
[Out] 1/(a*cos(x)^2)^(1/2)*a*cos(x)*sin(x)
```

Maxima [A] time = 2.32193, size = 8, normalized size = 0.62

$$\sqrt{a}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*sin(x)
```

Fricas [A] time = 1.0715, size = 43, normalized size = 3.31

$$\frac{\sqrt{a\cos(x)^2}\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

Sympy [A] time = 0.576638, size = 19, normalized size = 1.46

$$\frac{\sqrt{a}\sqrt{\cos^2(x)}\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**2)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(cos(x)**2)*sin(x)/cos(x)
```

Giac [A] time = 1.23228, size = 12, normalized size = 0.92

$$\sqrt{a}\operatorname{sgn}(\cos(x))\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(1/2),x, algorithm="giac")

[Out] sqrt(a)*sgn(cos(x))*sin(x)
```

3.42 $\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

Rubi [A] time = 0.0140495, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3770}

$$\frac{\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[x]^2],x]
```

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx &= \frac{\cos(x)\int \sec(x)\,dx}{\sqrt{a\cos^2(x)}}\\ &= \frac{\tanh^{-1}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}\end{aligned}$$

Mathematica [B] time = 0.0175431, size = 46, normalized size = 2.88

$$\frac{\cos(x)\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[x]^2],x]
```

```
[Out] (Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]))/Sqrt[a*Cos[
x]^2]
```

Maple [B] time = 0.708, size = 48, normalized size = 3.

$$\frac{\cos(x)}{\sin(x)}\sqrt{a(\sin(x))^2}\ln\left(2\frac{\sqrt{a}\sqrt{a(\sin(x))^2}+a}{\cos(x)}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{a(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^2)^(1/2),x)
```

```
[Out] cos(x)*(a*sin(x)^2)^(1/2)/a^(1/2)*ln(2/cos(x)*(a^(1/2)*(a*sin(x)^2)^(1/2)+a
))/sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [B] time = 2.67617, size = 51, normalized size = 3.19

$$\frac{\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1))/sqrt(a)
```

Fricas [B] time = 1.17792, size = 182, normalized size = 11.38

$$\left[-\frac{\sqrt{a\cos(x)^2}\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)}{2a\cos(x)},-\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos(x)^2}\sqrt{-a}\sin(x)}{a\cos(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a*cos(x)^2)*log(-(sin(x) - 1)/(sin(x) + 1))/(a*cos(x)), -sqrt(-a
)*arctan(sqrt(a*cos(x)^2)*sqrt(-a)*sin(x)/(a*cos(x)))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos{(x)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cos(x)^2), x)
```

3.43 $\int \frac{1}{\left(a \cos^2(x)\right)^{3/2}} \, dx$

Optimal. Leaf size=42

$$\frac{\tan(x)}{2a\sqrt{a \cos^2(x)}} + \frac{\cos(x) \tanh^{-1}(\sin(x))}{2a\sqrt{a \cos^2(x)}}$$

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x]
^2])
```

Rubi [A] time = 0.0223124, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3204, 3207, 3770}

$$\frac{\tan(x)}{2a\sqrt{a \cos^2(x)}} + \frac{\cos(x) \tanh^{-1}(\sin(x))}{2a\sqrt{a \cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^2)^(-3/2),x]
```

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x]
^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx = \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx}{2a}$$
$$= \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\cos(x)\int \sec(x)\,dx}{2a\sqrt{a\cos^2(x)}}$$
$$= \frac{\tanh^{-1}(\sin(x))\cos(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

Mathematica [B] time = 0.0640078, size = 91, normalized size = 2.17

$$-\frac{\cos(x)\left(-2\sin(x) + \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \cos(2x)\left(\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right.\right.}{4\left(a\cos^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^2)^(-3/2),x]
```

```
[Out] -(Cos[x]*(Log[Cos[x/2] - Sin[x/2]] + Cos[2*x]*(Log[Cos[x/2] - Sin[x/2]] - L
og[Cos[x/2] + Sin[x/2]]) - Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x]))/(4*(a*Cos[
x]^2)^(3/2))
```

Maple [B] time = 1.105, size = 70, normalized size = 1.7

$$\frac{1}{2\cos(x)\sin(x)}\sqrt{a(\sin(x))^2}\left(\ln\left(2\,\frac{\sqrt{a}\sqrt{a(\sin(x))^2}+a}{\cos(x)}\right)(\cos(x))^2 a + \sqrt{a}\sqrt{a(\sin(x))^2}\right)a^{-\frac{5}{2}}\frac{1}{\sqrt{a(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^2)^(3/2),x)
```

```
[Out] 1/2/a^(5/2)/cos(x)*(a*sin(x)^2)^(1/2)*(ln(2/cos(x)*(a^(1/2)*(a*sin(x)^2)^(1
/2)+a))*cos(x)^2*a+a^(1/2)*(a*sin(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [B] time = 2.33001, size = 410, normalized size = 9.76

$$\frac{4(\sin(3x) - \sin(x))\cos(4x) + \left(2(2\cos(2x) + 1)\cos(4x) + \cos(4x)^2 + 4\cos(2x)^2 + \sin(4x)^2 + 4\sin(4x)\sin(2x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*x) - sin(x))*cos(4*x) + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*
x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*c
os(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (2*(2*cos(2*x) + 1)*
cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4
*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(
```

```
cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x)*sin(
2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))/((a*cos(4*x)^2 + 4
*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)^2 + 2*(
2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*sqrt(a))
```

Fricas [A] time = 1.14468, size = 124, normalized size = 2.95

$$-\frac{\sqrt{a \cos\left(x\right)^{2}}\left(\cos\left(x\right)^{2} \log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right) - 2 \, \sin\left(x\right)\right)}{4 \, a^{2} \cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(x)^2)*(cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*sin(x))
/(a^2*cos(x)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.78857, size = 59, normalized size = 1.4

$$-\frac{\sqrt{a} \log\left({\left| -\sqrt{a} \tan\left(x\right) + \sqrt{a \tan\left(x\right)^{2} + a} \right|}\right) - \sqrt{a \tan\left(x\right)^{2} + a} \tan\left(x\right)}{2 \, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(a)*log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a))) - sqrt(a*tan
(x)^2 + a)*tan(x))/a^2
```

3.44 $\int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{3\cos(x)\tanh^{-1}(\sin(x))}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x])/(8*a^2*Sqrt[a*Cos[x]^2]) + Tan[x]/(4*a*(a*Cos[x]
^2)^(3/2)) + (3*Tan[x])/(8*a^2*Sqrt[a*Cos[x]^2])
```

Rubi [A] time = 0.0437965, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3204, 3207, 3770}

$$\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{3\cos(x)\tanh^{-1}(\sin(x))}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^2)^(-5/2),x]
```

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x])/(8*a^2*Sqrt[a*Cos[x]^2]) + Tan[x]/(4*a*(a*Cos[x]
^2)^(3/2)) + (3*Tan[x])/(8*a^2*Sqrt[a*Cos[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx = \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx}{4a}$$

$$= \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}} + \frac{3\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx}{8a^2}$$

$$= \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}} + \frac{(3\cos(x))\int \sec(x)\,dx}{8a^2\sqrt{a\cos^2(x)}}$$

$$= \frac{3\tanh^{-1}(\sin(x))\cos(x)}{8a^2\sqrt{a\cos^2(x)}} + \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

Mathematica [A] time = 0.156865, size = 72, normalized size = 1.18

$$\frac{\cos^5(x)\left(\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x)-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{16\left(a\cos^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^2)^(-5/2),x]
```

```
[Out] (Cos[x]^5*(-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[
x]^4*(11*Sin[x] + 3*Sin[3*x]))/2))/(16*(a*Cos[x]^2)^(5/2))
```

Maple [A] time = 1.112, size = 89, normalized size = 1.5

$$\frac{1}{8\,(\cos(x))^3\sin(x)}\sqrt{a(\sin(x))^2}\left(3\ln\left(2\frac{\sqrt{a}\sqrt{a(\sin(x))^2}+a}{\cos(x)}\right)(\cos(x))^4a+3\sqrt{a(\sin(x))^2}(\cos(x))^2\sqrt{a}+2\sqrt{a}\sqrt{a(s$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^2)^(5/2),x)
```

```
[Out] 1/8/a^(7/2)/cos(x)^3*(a*sin(x)^2)^(1/2)*(3*ln(2/cos(x)*(a^(1/2)*(a*sin(x)^2
)^(1/2)+a))*cos(x)^4*a+3*(a*sin(x)^2)^(1/2)*cos(x)^2*a^(1/2)+2*a^(1/2)*(a*s
in(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [B] time = 3.66056, size = 1260, normalized size = 20.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*sin(7*x) + 11*sin(5*x) - 11*sin(3*x) - 3*sin(x))*cos(8*x) - 24*(
2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 16*(11*sin(5*x) - 11*sin(3
```

```
*x) - 3*sin(x))*cos(6*x) - 88*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) - 24*(11*s
in(3*x) + 3*sin(x))*cos(4*x) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) +
 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*c
os(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 +
4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(
4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*s
in(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x
) + 1) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x
)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(
2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(
4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin
(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^
2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(3*cos(7*x)
 + 11*cos(5*x) - 11*cos(3*x) - 3*cos(x))*sin(8*x) + 12*(4*cos(6*x) + 6*cos(
4*x) + 4*cos(2*x) + 1)*sin(7*x) - 16*(11*cos(5*x) - 11*cos(3*x) - 3*cos(x))
*sin(6*x) + 44*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) + 24*(11*cos(3*x) + 3
*cos(x))*sin(4*x) - 44*(4*cos(2*x) + 1)*sin(3*x) + 176*cos(3*x)*sin(2*x) +
48*cos(x)*sin(2*x) - 48*cos(2*x)*sin(x) - 12*sin(x))/((a^2*cos(8*x)^2 + 16*
a^2*cos(6*x)^2 + 36*a^2*cos(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 1
6*a^2*sin(6*x)^2 + 36*a^2*sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*si
n(2*x)^2 + 8*a^2*cos(2*x) + a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^
2*cos(2*x) + a^2)*cos(8*x) + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(
6*x) + 12*(4*a^2*cos(2*x) + a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4
*x) + 2*a^2*sin(2*x))*sin(8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6
*x))*sqrt(a))
```

Fricas [A] time = 1.14584, size = 151, normalized size = 2.48

$$-\frac{\left(3\,\cos\left(x\right)^{4}\log\left(-\frac{\sin\left(x\right)-1}{\sin\left(x\right)+1}\right)-2\left(3\,\cos\left(x\right)^{2}+2\right)\sin\left(x\right)\right)\sqrt{a\cos\left(x\right)^{2}}}{16\,a^{3}\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*cos(x)^2 + 2)*sin(
x))*sqrt(a*cos(x)^2)/(a^3*cos(x)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.56893, size = 170, normalized size = 2.79

$$\frac{\frac{3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)+2\right|\right)}{\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2+1\right)}-\frac{3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)-2\right|\right)}{\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2+1\right)}+\frac{4\left(5\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^3-\frac{12}{\tan\left(\frac{1}{2}x\right)}-12\tan\left(\frac{1}{2}x\right)\right)}{\left(\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^2-4\right)^2\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2+1\right)}}{16a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*log(abs(1/tan(1/2*x) + tan(1/2*x) + 2))/sgn(-tan(1/2*x)^2 + 1) - 3*
log(abs(1/tan(1/2*x) + tan(1/2*x) - 2))/sgn(-tan(1/2*x)^2 + 1) + 4*(5*(1/ta
n(1/2*x) + tan(1/2*x))^3 - 12/tan(1/2*x) - 12*tan(1/2*x))/(((1/tan(1/2*x) +
 tan(1/2*x))^2 - 4)^2*sgn(-tan(1/2*x)^2 + 1)))/a^(5/2)
```

3.45 $\int \left(a \cos^3(x)\right)^{5/2} dx$

Optimal. Leaf size=117

$$\frac{2}{15}a^2\sin(x)\cos^5(x)\sqrt{a\cos^3(x)}+\frac{26}{165}a^2\sin(x)\cos^3(x)\sqrt{a\cos^3(x)}+\frac{78}{385}a^2\sin(x)\cos(x)\sqrt{a\cos^3(x)}+\frac{26}{77}a^2\tan(x)\sqrt{a\cos^3(x)}$$

```
[Out] (26*a^2*Sqrt[a*Cos[x]^3]*EllipticF[x/2, 2])/(77*Cos[x]^(3/2)) + (78*a^2*Cos
[x]*Sqrt[a*Cos[x]^3]*Sin[x])/385 + (26*a^2*Cos[x]^3*Sqrt[a*Cos[x]^3]*Sin[x]
)/165 + (2*a^2*Cos[x]^5*Sqrt[a*Cos[x]^3]*Sin[x])/15 + (26*a^2*Sqrt[a*Cos[x]
^3]*Tan[x])/77
```

Rubi [A] time = 0.065901, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 2641}

$$\frac{2}{15}a^2\sin(x)\cos^5(x)\sqrt{a\cos^3(x)}+\frac{26}{165}a^2\sin(x)\cos^3(x)\sqrt{a\cos^3(x)}+\frac{78}{385}a^2\sin(x)\cos(x)\sqrt{a\cos^3(x)}+\frac{26}{77}a^2\tan(x)\sqrt{a\cos^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^3)^(5/2),x]
```



```
[Out] (26*a^2*Sqrt[a*Cos[x]^3]*EllipticF[x/2, 2])/(77*Cos[x]^(3/2)) + (78*a^2*Cos
[x]*Sqrt[a*Cos[x]^3]*Sin[x])/385 + (26*a^2*Cos[x]^3*Sqrt[a*Cos[x]^3]*Sin[x]
)/165 + (2*a^2*Cos[x]^5*Sqrt[a*Cos[x]^3]*Sin[x])/15 + (26*a^2*Sqrt[a*Cos[x]
^3]*Tan[x])/77
```


Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\cos^3(x)\right)^{5/2} dx = \frac{\left(a^2\sqrt{a\cos^3(x)}\right)\int \cos^{\frac{15}{2}}(x)\,dx}{\cos^{\frac{3}{2}}(x)}$$

$$= \frac{2}{15}a^2\cos^5(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{\left(13a^2\sqrt{a\cos^3(x)}\right)\int \cos^{\frac{11}{2}}(x)\,dx}{15\cos^{\frac{3}{2}}(x)}$$

$$= \frac{26}{165}a^2\cos^3(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{2}{15}a^2\cos^5(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{\left(39a^2\sqrt{a\cos^3(x)}\right)\int \cos^{\frac{7}{2}}(x}{55\cos^{\frac{3}{2}}(x)}$$

$$= \frac{78}{385}a^2\cos(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{26}{165}a^2\cos^3(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{2}{15}a^2\cos^5(x)\sqrt{a\cos^3(x)}\sin$$

$$= \frac{78}{385}a^2\cos(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{26}{165}a^2\cos^3(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{2}{15}a^2\cos^5(x)\sqrt{a\cos^3(x)}\sin$$

$$= \frac{26a^2\sqrt{a\cos^3(x)}F\left(\frac{x}{2}\middle|2\right)}{77\cos^{\frac{3}{2}}(x)} + \frac{78}{385}a^2\cos(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{26}{165}a^2\cos^3(x)\sqrt{a\cos^3(x)}\sin(x) +$$

Mathematica [A] time = 0.1198, size = 61, normalized size = 0.52

$$\frac{a\left(a\cos^3(x)\right)^{3/2}\left(12480F\left(\frac{x}{2}\middle|2\right) + (15465\sin(x) + 3657\sin(3x) + 749\sin(5x) + 77\sin(7x))\sqrt{\cos(x)}\right)}{36960\cos^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^3)^(5/2),x]
```

```
[Out] (a*(a*Cos[x]^3)^(3/2)*(12480*EllipticF[x/2, 2] + Sqrt[Cos[x]]*(15465*Sin[x]
 + 3657*Sin[3*x] + 749*Sin[5*x] + 77*Sin[7*x])))/(36960*Cos[x]^(9/2))
```

Maple [C] time = 0.301, size = 114, normalized size = 1.

$$-\frac{(-2+2\cos(x))(\cos(x)+1)^2}{1155(\sin(x))^3(\cos(x))^8}\left(-77(\cos(x))^8 + 77(\cos(x))^7 - 91(\cos(x))^6 + 195\,i\sqrt{(\cos(x)+1)^{-1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^3)^(5/2),x)
```

```
[Out] -2/1155*(-1+cos(x))*(-77*cos(x)^8+77*cos(x)^7-91*cos(x)^6+195*I*(1/(cos(x)+
1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)*sin(x
)+91*cos(x)^5-117*cos(x)^4+117*cos(x)^3-195*cos(x)^2+195*cos(x))*(cos(x)+1)
^2*(a*cos(x)^3)^(5/2)/sin(x)^3/cos(x)^8
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos(x)^3\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(x)^3)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\cos(x)^3}a^2\cos(x)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x)^3)*a^2*cos(x)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos(x)^3\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(x)^3)^(5/2), x)
```

3.46 $\int \left(a\cos^3(x)\right)^{3/2} dx$

Optimal. Leaf size=67

$$\frac{2}{9}a\sin(x)\cos^2(x)\sqrt{a\cos^3(x)}+\frac{14}{45}a\sin(x)\sqrt{a\cos^3(x)}+\frac{14aE\left(\frac{x}{2}\middle|2\right)\sqrt{a\cos^3(x)}}{15\cos^{\frac{3}{2}}(x)}$$

```
[Out] (14*a*Sqrt[a*Cos[x]^3]*EllipticE[x/2, 2])/(15*Cos[x]^(3/2)) + (14*a*Sqrt[a*
Cos[x]^3]*Sin[x])/45 + (2*a*Cos[x]^2*Sqrt[a*Cos[x]^3]*Sin[x])/9
```

Rubi [A] time = 0.0408263, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 2639}

$$\frac{2}{9}a\sin(x)\cos^2(x)\sqrt{a\cos^3(x)}+\frac{14}{45}a\sin(x)\sqrt{a\cos^3(x)}+\frac{14aE\left(\frac{x}{2}\middle|2\right)\sqrt{a\cos^3(x)}}{15\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^3)^(3/2),x]
```

```
[Out] (14*a*Sqrt[a*Cos[x]^3]*EllipticE[x/2, 2])/(15*Cos[x]^(3/2)) + (14*a*Sqrt[a*
Cos[x]^3]*Sin[x])/45 + (2*a*Cos[x]^2*Sqrt[a*Cos[x]^3]*Sin[x])/9
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\cos^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\cos^3(x)}\right)\int \cos^{\frac{9}{2}}(x)\,dx}{\cos^{\frac{3}{2}}(x)}$$
$$= \frac{2}{9}a\cos^2(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{\left(7a\sqrt{a\cos^3(x)}\right)\int \cos^{\frac{5}{2}}(x)\,dx}{9\cos^{\frac{3}{2}}(x)}$$
$$= \frac{14}{45}a\sqrt{a\cos^3(x)}\sin(x) + \frac{2}{9}a\cos^2(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{\left(7a\sqrt{a\cos^3(x)}\right)\int \sqrt{\cos(x)}\,dx}{15\cos^{\frac{3}{2}}(x)}$$
$$= \frac{14a\sqrt{a\cos^3(x)}E\left(\frac{x}{2}\middle|2\right)}{15\cos^{\frac{3}{2}}(x)} + \frac{14}{45}a\sqrt{a\cos^3(x)}\sin(x) + \frac{2}{9}a\cos^2(x)\sqrt{a\cos^3(x)}\sin(x)$$

Mathematica [A] time = 0.0645819, size = 50, normalized size = 0.75

$$\frac{\left(a\cos^3(x)\right)^{3/2}\left(168E\left(\frac{x}{2}\middle|2\right) + (38\sin(2x) + 5\sin(4x))\sqrt{\cos(x)}\right)}{180\cos^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^3)^(3/2),x]
```

```
[Out] ((a*Cos[x]^3)^(3/2)*(168*EllipticE[x/2, 2] + Sqrt[Cos[x]]*(38*Sin[2*x] + 5*
Sin[4*x])))/(180*Cos[x]^(9/2))
```

Maple [C] time = 0.254, size = 198, normalized size = 3.

$$-\frac{2}{45\,(\cos(x))^5\sin(x)}\left(5\,(\cos(x))^6 + 21\,i\cos(x)\sin(x)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(x))}{\sin(x)}, i\right)\sqrt{(\cos(x)+1)^{-1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}} - 21\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^3)^(3/2),x)
```

```
[Out] -2/45*(5*cos(x)^6+21*I*cos(x)*sin(x)*EllipticE(I*(-1+cos(x))/sin(x),I)*(1/(
cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)-21*I*cos(x)*sin(x)*EllipticF(I*(
-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)+21*I*si
n(x)*EllipticE(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)
+1))^(1/2)-21*I*sin(x)*EllipticF(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/
2)*(cos(x)/(cos(x)+1))^(1/2)+2*cos(x)^4+14*cos(x)^2-21*cos(x))*(a*cos(x)^3)
^(3/2)/cos(x)^5/sin(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(x)^3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\cos\left(x\right)^3}a\cos\left(x\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x)^3)*a*cos(x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**3)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(x\right)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(x)^3)^(3/2), x)
```

3.47 $\int \sqrt{a\cos^3(x)}\,dx$

Optimal. Leaf size=44

$$\frac{2}{3}\tan(x)\sqrt{a\cos^3(x)}+\frac{2F\left(\frac{x}{2}\middle|2\right)\sqrt{a\cos^3(x)}}{3\cos^{\frac{3}{2}}(x)}$$

```
[Out] (2*Sqrt[a*Cos[x]^3]*EllipticF[x/2, 2])/(3*Cos[x]^(3/2)) + (2*Sqrt[a*Cos[x]^
3]*Tan[x])/3
```

Rubi [A] time = 0.0283456, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 2641}

$$\frac{2}{3}\tan(x)\sqrt{a\cos^3(x)}+\frac{2F\left(\frac{x}{2}\middle|2\right)\sqrt{a\cos^3(x)}}{3\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[x]^3],x]
```

```
[Out] (2*Sqrt[a*Cos[x]^3]*EllipticF[x/2, 2])/(3*Cos[x]^(3/2)) + (2*Sqrt[a*Cos[x]^
3]*Tan[x])/3
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\cos^3(x)}\,dx = \frac{\sqrt{a\cos^3(x)}\int \cos^{\frac{3}{2}}(x)\,dx}{\cos^{\frac{3}{2}}(x)}$$
$$= \frac{2}{3}\sqrt{a\cos^3(x)}\tan(x) + \frac{\sqrt{a\cos^3(x)}\int \frac{1}{\sqrt{\cos(x)}}\,dx}{3\cos^{\frac{3}{2}}(x)}$$
$$= \frac{2\sqrt{a\cos^3(x)}F\left(\frac{x}{2}\middle|2\right)}{3\cos^{\frac{3}{2}}(x)} + \frac{2}{3}\sqrt{a\cos^3(x)}\tan(x)$$

Mathematica [A] time = 0.0254141, size = 37, normalized size = 0.84

$$\frac{2\sqrt{a\cos^3(x)}\left(F\left(\frac{x}{2}\middle|2\right)+\sin(x)\sqrt{\cos(x)}\right)}{3\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[x]^3],x]
```

```
[Out] (2*Sqrt[a*Cos[x]^3]*(EllipticF[x/2, 2] + Sqrt[Cos[x]]*Sin[x]))/(3*Cos[x]^(3
/2))
```

Maple [C] time = 0.262, size = 76, normalized size = 1.7

$$\frac{(-2+2\cos(x))(\cos(x)+1)^2}{3(\cos(x))^2(\sin(x))^3}\left(-i\sqrt{(\cos(x)+1)^{-1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)\sin(x)+(\cos(x))^2-\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^3)^(1/2),x)
```

```
[Out] 2/3*(-1+cos(x))*(-I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*Elliptic
F(I*(-1+cos(x))/sin(x),I)*sin(x)+cos(x)^2-cos(x))*(cos(x)+1)^2*(a*cos(x)^3)
^(1/2)/cos(x)^2/sin(x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cos(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\cos(x)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**3)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(x)^3), x)
```

3.48 $\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sin(x)\cos(x)}{\sqrt{a\cos^3(x)}}-\frac{2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{\sqrt{a\cos^3(x)}}$$

```
[Out] (-2*Cos[x]^(3/2)*EllipticE[x/2, 2])/Sqrt[a*Cos[x]^3] + (2*Cos[x]*Sin[x])/Sq
rt[a*Cos[x]^3]
```

Rubi [A] time = 0.0234349, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2636, 2639}

$$\frac{2\sin(x)\cos(x)}{\sqrt{a\cos^3(x)}}-\frac{2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{\sqrt{a\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[x]^3],x]
```

```
[Out] (-2*Cos[x]^(3/2)*EllipticE[x/2, 2])/Sqrt[a*Cos[x]^3] + (2*Cos[x]*Sin[x])/Sq
rt[a*Cos[x]^3]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx = \frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\cos^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\cos^3(x)}}$$
$$= \frac{2\cos(x)\sin(x)}{\sqrt{a\cos^3(x)}} - \frac{\cos^{\frac{3}{2}}(x)\int \sqrt{\cos(x)}\,dx}{\sqrt{a\cos^3(x)}}$$
$$= -\frac{2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{\sqrt{a\cos^3(x)}} + \frac{2\cos(x)\sin(x)}{\sqrt{a\cos^3(x)}}$$

Mathematica [A] time = 0.0210661, size = 31, normalized size = 0.74

$$\frac{\sin(2x) - 2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{\sqrt{a\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[x]^3],x]
```

```
[Out] (-2*Cos[x]^(3/2)*EllipticE[x/2, 2] + Sin[2*x])/Sqrt[a*Cos[x]^3]
```

Maple [C] time = 0.333, size = 191, normalized size = 4.6

$$2\,\frac{(\cos(x)+1)^2(-1+\cos(x))^2\cos(x)}{\sqrt{a(\cos(x))^3}(\sin(x))^5}\left(i\cos(x)\sin(x)\sqrt{(\cos(x)+1)^{-1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\mathrm{EllipticE}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right) - i\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^3)^(1/2),x)
```

```
[Out] 2*(cos(x)+1)^2*(-1+cos(x))^2*(I*cos(x)*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/
(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(x))/sin(x),I)-I*cos(x)*sin(x)*(1/(cos
(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)+I
*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(
x))/sin(x),I)-I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*
(-1+cos(x))/sin(x),I)*sin(x)-cos(x)+1)*cos(x)/(a*cos(x)^3)^(1/2)/sin(x)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*cos(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cos\left(x\right)^{3}}}{a\cos\left(x\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x)^3)/(a*cos(x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**3)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos\left(x\right)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cos(x)^3), x)
```

3.49 $\int \frac{1}{\left(a\cos^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{10\sin(x)}{21a\sqrt{a\cos^3(x)}}+\frac{10\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)}{21a\sqrt{a\cos^3(x)}}+\frac{2\tan(x)\sec(x)}{7a\sqrt{a\cos^3(x)}}$$

```
[Out] (10*Cos[x]^(3/2)*EllipticF[x/2, 2])/(21*a*Sqrt[a*Cos[x]^3]) + (10*Sin[x])/(
21*a*Sqrt[a*Cos[x]^3]) + (2*Sec[x]*Tan[x])/(7*a*Sqrt[a*Cos[x]^3])
```

Rubi [A] time = 0.0303169, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2636, 2641}

$$\frac{10\sin(x)}{21a\sqrt{a\cos^3(x)}}+\frac{10\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)}{21a\sqrt{a\cos^3(x)}}+\frac{2\tan(x)\sec(x)}{7a\sqrt{a\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^3)^(-3/2),x]
```

```
[Out] (10*Cos[x]^(3/2)*EllipticF[x/2, 2])/(21*a*Sqrt[a*Cos[x]^3]) + (10*Sin[x])/(
21*a*Sqrt[a*Cos[x]^3]) + (2*Sec[x]*Tan[x])/(7*a*Sqrt[a*Cos[x]^3])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\cos^3(x)\right)^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\cos^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\cos^3(x)}}$$

$$= \frac{2\sec(x)\tan(x)}{7a\sqrt{a\cos^3(x)}} + \frac{\left(5\cos^{\frac{3}{2}}(x)\right)\int \frac{1}{\cos^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\cos^3(x)}}$$

$$= \frac{10\sin(x)}{21a\sqrt{a\cos^3(x)}} + \frac{2\sec(x)\tan(x)}{7a\sqrt{a\cos^3(x)}} + \frac{\left(5\cos^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\cos(x)}}\,dx}{21a\sqrt{a\cos^3(x)}}$$

$$= \frac{10\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)}{21a\sqrt{a\cos^3(x)}} + \frac{10\sin(x)}{21a\sqrt{a\cos^3(x)}} + \frac{2\sec(x)\tan(x)}{7a\sqrt{a\cos^3(x)}}$$

Mathematica [A] time = 0.0592409, size = 44, normalized size = 0.62

$$\frac{2\cos^2(x)\left(3\tan(x)+5\cos^{\frac{5}{2}}(x)F\left(\frac{x}{2}\middle|2\right)+5\sin(x)\cos(x)\right)}{21\left(a\cos^3(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^3)^(-3/2),x]
```

```
[Out] (2*Cos[x]^2*(5*Cos[x]^(5/2)*EllipticF[x/2, 2] + 5*Cos[x]*Sin[x] + 3*Tan[x])
)/(21*(a*Cos[x]^3)^(3/2))
```

Maple [C] time = 0.289, size = 87, normalized size = 1.2

$$-\frac{2\,(\cos(x)+1)^2\,(-1+\cos(x))\cos(x)}{21\,(\sin(x))^3}\left(5\,i\,(\cos(x))^3\sin(x)\,\mathrm{EllipticF}\left(\frac{i\,(-1+\cos(x))}{\sin(x)},i\right)\sqrt{(\cos(x)+1)^{-1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^3)^(3/2),x)
```

```
[Out] -2/21*(cos(x)+1)^2*(-1+cos(x))*(5*I*cos(x)^3*sin(x)*EllipticF(I*(-1+cos(x))
/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)-5*cos(x)^3+5*cos(
x)^2-3*cos(x)+3)*cos(x)/sin(x)^3/(a*cos(x)^3)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(x)^3)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cos\left(x\right)^{3}}}{a^{2}\cos\left(x\right)^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x)^3)/(a^2*cos(x)^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**3)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(x\right)^{3}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(x)^3)^(-3/2), x)
```

3.50 $\int \frac{1}{\left(a\cos^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{154\sin(x)\cos(x)}{195a^2\sqrt{a\cos^3(x)}}+\frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}}-\frac{154\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\cos^3(x)}}+\frac{2\tan(x)\sec^4(x)}{13a^2\sqrt{a\cos^3(x)}}+\frac{22\tan(x)\sec^2(x)}{117a^2\sqrt{a\cos^3(x)}}$$

```
[Out] (-154*Cos[x]^(3/2)*EllipticE[x/2, 2])/(195*a^2*Sqrt[a*Cos[x]^3]) + (154*Cos
[x]*Sin[x])/(195*a^2*Sqrt[a*Cos[x]^3]) + (154*Tan[x])/(585*a^2*Sqrt[a*Cos[x
]^3]) + (22*Sec[x]^2*Tan[x])/(117*a^2*Sqrt[a*Cos[x]^3]) + (2*Sec[x]^4*Tan[x
])/(13*a^2*Sqrt[a*Cos[x]^3])
```

Rubi [A] time = 0.0509484, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2636, 2639}

$$\frac{154\sin(x)\cos(x)}{195a^2\sqrt{a\cos^3(x)}}+\frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}}-\frac{154\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\cos^3(x)}}+\frac{2\tan(x)\sec^4(x)}{13a^2\sqrt{a\cos^3(x)}}+\frac{22\tan(x)\sec^2(x)}{117a^2\sqrt{a\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^3)^(-5/2),x]
```

```
[Out] (-154*Cos[x]^(3/2)*EllipticE[x/2, 2])/(195*a^2*Sqrt[a*Cos[x]^3]) + (154*Cos
[x]*Sin[x])/(195*a^2*Sqrt[a*Cos[x]^3]) + (154*Tan[x])/(585*a^2*Sqrt[a*Cos[x
]^3]) + (22*Sec[x]^2*Tan[x])/(117*a^2*Sqrt[a*Cos[x]^3]) + (2*Sec[x]^4*Tan[x
])/(13*a^2*Sqrt[a*Cos[x]^3])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\cos^3(x)\right)^{5/2}}\,dx = \frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\cos^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\cos^3(x)}}$$

$$= \frac{2\sec^4(x)\tan(x)}{13a^2\sqrt{a\cos^3(x)}} + \frac{\left(11\cos^{\frac{3}{2}}(x)\right)\int \frac{1}{\cos^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\cos^3(x)}}$$

$$= \frac{22\sec^2(x)\tan(x)}{117a^2\sqrt{a\cos^3(x)}} + \frac{2\sec^4(x)\tan(x)}{13a^2\sqrt{a\cos^3(x)}} + \frac{\left(77\cos^{\frac{3}{2}}(x)\right)\int \frac{1}{\cos^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\cos^3(x)}}$$

$$= \frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}} + \frac{22\sec^2(x)\tan(x)}{117a^2\sqrt{a\cos^3(x)}} + \frac{2\sec^4(x)\tan(x)}{13a^2\sqrt{a\cos^3(x)}} + \frac{\left(77\cos^{\frac{3}{2}}(x)\right)\int \frac{1}{\cos^{\frac{3}{2}}(x)}\,dx}{195a^2\sqrt{a\cos^3(x)}}$$

$$= \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\cos^3(x)}} + \frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}} + \frac{22\sec^2(x)\tan(x)}{117a^2\sqrt{a\cos^3(x)}} + \frac{2\sec^4(x)\tan(x)}{13a^2\sqrt{a\cos^3(x)}} - \frac{\left(77\cos^{\frac{3}{2}}(x)\right)\int \sqrt{\cdots}}{195a^2\sqrt{a\cos\cdots}}$$

$$= -\frac{154\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\cos^3(x)}} + \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\cos^3(x)}} + \frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}} + \frac{22\sec^2(x)\tan(x)}{117a^2\sqrt{a\cos^3(x)}} + \frac{2\sec^4(x)\tan\cdots}{13a^2\sqrt{a\cos\cdots}}$$

Mathematica [A] time = 0.104767, size = 57, normalized size = 0.49

$$\frac{-462\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right) + 462\sin(x)\cos(x) + 2\tan(x)\left(45\sec^4(x) + 55\sec^2(x) + 77\right)}{585a^2\sqrt{a\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^3)^(-5/2),x]
```

```
[Out] (-462*Cos[x]^(3/2)*EllipticE[x/2, 2] + 462*Cos[x]*Sin[x] + 2*(77 + 55*Sec[x
]^2 + 45*Sec[x]^4)*Tan[x])/(585*a^2*Sqrt[a*Cos[x]^3])
```

Maple [C] time = 0.423, size = 223, normalized size = 1.9

$$-\frac{2\,(\cos(x)+1)^2\,(-1+\cos(x))^2\cos(x)}{585\,(\sin(x))^5}\left(231\,i\,(\cos(x))^7\sin(x)\sqrt{(\cos(x)+1)^{-1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^3)^(5/2),x)
```

```
[Out] -2/585*(cos(x)+1)^2*(-1+cos(x))^2*(231*I*cos(x)^7*sin(x)*(1/(cos(x)+1))^(1/
2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-231*I*cos(x)
^7*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+co
s(x))/sin(x),I)+231*I*cos(x)^6*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+
1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-231*I*cos(x)^6*sin(x)*(1/(cos(x
)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(x))/sin(x),I)+231
*cos(x)^7-154*cos(x)^6-22*cos(x)^4-10*cos(x)^2-45)*cos(x)/sin(x)^5/(a*cos(x
)^3)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos{(x)}^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(x)^3)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cos{(x)}^3}}{a^3\cos{(x)}^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cos(x)^3)/(a^3*cos(x)^9), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos{(x)}^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(x)^3)^(-5/2), x)
```

3.51 $\int \left(a\cos^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=132

$$\frac{1}{10}a^2\sin(x)\cos^7(x)\sqrt{a\cos^4(x)}+\frac{9}{80}a^2\sin(x)\cos^5(x)\sqrt{a\cos^4(x)}+\frac{21}{160}a^2\sin(x)\cos^3(x)\sqrt{a\cos^4(x)}+\frac{21}{128}a^2\sin(x)\cos(x)$$

```
[Out] (63*a^2*x*Sqrt[a*Cos[x]^4]*Sec[x]^2)/256 + (21*a^2*Cos[x]*Sqrt[a*Cos[x]^4]*
Sin[x])/128 + (21*a^2*Cos[x]^3*Sqrt[a*Cos[x]^4]*Sin[x])/160 + (9*a^2*Cos[x]
^5*Sqrt[a*Cos[x]^4]*Sin[x])/80 + (a^2*Cos[x]^7*Sqrt[a*Cos[x]^4]*Sin[x])/10
+ (63*a^2*Sqrt[a*Cos[x]^4]*Tan[x])/256
```

Rubi [A] time = 0.0510577, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$\frac{1}{10}a^2\sin(x)\cos^7(x)\sqrt{a\cos^4(x)}+\frac{9}{80}a^2\sin(x)\cos^5(x)\sqrt{a\cos^4(x)}+\frac{21}{160}a^2\sin(x)\cos^3(x)\sqrt{a\cos^4(x)}+\frac{21}{128}a^2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^4)^(5/2),x]
```

```
[Out] (63*a^2*x*Sqrt[a*Cos[x]^4]*Sec[x]^2)/256 + (21*a^2*Cos[x]*Sqrt[a*Cos[x]^4]*
Sin[x])/128 + (21*a^2*Cos[x]^3*Sqrt[a*Cos[x]^4]*Sin[x])/160 + (9*a^2*Cos[x]
^5*Sqrt[a*Cos[x]^4]*Sin[x])/80 + (a^2*Cos[x]^7*Sqrt[a*Cos[x]^4]*Sin[x])/10
+ (63*a^2*Sqrt[a*Cos[x]^4]*Tan[x])/256
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a\cos^4(x)\right)^{5/2} dx = \left(a^2\sqrt{a\cos^4(x)}\sec^2(x)\right)\int \cos^{10}(x)\,dx$$
$$= \frac{1}{10}a^2\cos^7(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{10}\left(9a^2\sqrt{a\cos^4(x)}\sec^2(x)\right)\int \cos^8(x)\,dx$$
$$= \frac{9}{80}a^2\cos^5(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{10}a^2\cos^7(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{80}\left(63a^2\sqrt{a\cos^4(x)}\sec^2(x)\right.$$
$$= \frac{21}{160}a^2\cos^3(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{9}{80}a^2\cos^5(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{10}a^2\cos^7(x)\sqrt{a\cos^4(x)}\sin$$
$$= \frac{21}{128}a^2\cos(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{21}{160}a^2\cos^3(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{9}{80}a^2\cos^5(x)\sqrt{a\cos^4(x)}\sin$$
$$= \frac{21}{128}a^2\cos(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{21}{160}a^2\cos^3(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{9}{80}a^2\cos^5(x)\sqrt{a\cos^4(x)}\sin$$
$$= \frac{63}{256}a^2x\sqrt{a\cos^4(x)}\sec^2(x) + \frac{21}{128}a^2\cos(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{21}{160}a^2\cos^3(x)\sqrt{a\cos^4(x)}\sin(x)$$

Mathematica [A] time = 0.123115, size = 53, normalized size = 0.4

$$\frac{a(2520x + 2100\sin(2x) + 600\sin(4x) + 150\sin(6x) + 25\sin(8x) + 2\sin(10x))\sec^6(x)\left(a\cos^4(x)\right)^{3/2}}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^4)^(5/2),x]
```

```
[Out] (a*(a*Cos[x]^4)^(3/2)*Sec[x]^6*(2520*x + 2100*Sin[2*x] + 600*Sin[4*x] + 150
*Sin[6*x] + 25*Sin[8*x] + 2*Sin[10*x]))/10240
```

Maple [A] time = 0.295, size = 57, normalized size = 0.4

$$\frac{128\sin(x)(\cos(x))^9 + 144\sin(x)(\cos(x))^7 + 168\sin(x)(\cos(x))^5 + 210(\cos(x))^3\sin(x) + 315\cos(x)\sin(x) + 31}{1280(\cos(x))^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^4)^(5/2),x)
```

```
[Out] 1/1280*(a*cos(x)^4)^(5/2)*(128*sin(x)*cos(x)^9+144*sin(x)*cos(x)^7+168*sin(
x)*cos(x)^5+210*cos(x)^3*sin(x)+315*cos(x)*sin(x)+315*x)/cos(x)^10
```

Maxima [A] time = 2.47314, size = 115, normalized size = 0.87

$$\frac{63}{256}a^{\frac{5}{2}}x + \frac{315\,a^{\frac{5}{2}}\tan(x)^9 + 1470\,a^{\frac{5}{2}}\tan(x)^7 + 2688\,a^{\frac{5}{2}}\tan(x)^5 + 2370\,a^{\frac{5}{2}}\tan(x)^3 + 965\,a^{\frac{5}{2}}\tan(x)}{1280\left(\tan(x)^{10} + 5\tan(x)^8 + 10\tan(x)^6 + 10\tan(x)^4 + 5\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 63/256*a^(5/2)*x + 1/1280*(315*a^(5/2)*tan(x)^9 + 1470*a^(5/2)*tan(x)^7 + 2
688*a^(5/2)*tan(x)^5 + 2370*a^(5/2)*tan(x)^3 + 965*a^(5/2)*tan(x))/(tan(x)^
```

```
10 + 5*tan(x)^8 + 10*tan(x)^6 + 10*tan(x)^4 + 5*tan(x)^2 + 1)
```

Fricas [A] time = 1.16141, size = 200, normalized size = 1.52

$$\frac{\sqrt{a \cos(x)^4}\left(315\, a^2 x + \left(128\, a^2 \cos(x)^9 + 144\, a^2 \cos(x)^7 + 168\, a^2 \cos(x)^5 + 210\, a^2 \cos(x)^3 + 315\, a^2 \cos(x)\right)\sin(x)\right)}{1280\, \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1280*sqrt(a*cos(x)^4)*(315*a^2*x + (128*a^2*cos(x)^9 + 144*a^2*cos(x)^7 +
 168*a^2*cos(x)^5 + 210*a^2*cos(x)^3 + 315*a^2*cos(x))*sin(x))/cos(x)^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39909, size = 77, normalized size = 0.58

$$\frac{1}{10240}\left(2520\, a^2 x + 2\, a^2 \sin(10\, x) + 25\, a^2 \sin(8\, x) + 150\, a^2 \sin(6\, x) + 600\, a^2 \sin(4\, x) + 2100\, a^2 \sin(2\, x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/10240*(2520*a^2*x + 2*a^2*sin(10*x) + 25*a^2*sin(8*x) + 150*a^2*sin(6*x)
+ 600*a^2*sin(4*x) + 2100*a^2*sin(2*x))*sqrt(a)
```

3.52 $\int \left(a\cos^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$\frac{1}{6}a\sin(x)\cos^3(x)\sqrt{a\cos^4(x)}+\frac{5}{24}a\sin(x)\cos(x)\sqrt{a\cos^4(x)}+\frac{5}{16}a\tan(x)\sqrt{a\cos^4(x)}+\frac{5}{16}ax\sec^2(x)\sqrt{a\cos^4(x)}$$

```
[Out] (5*a*x*Sqrt[a*Cos[x]^4]*Sec[x]^2)/16 + (5*a*Cos[x]*Sqrt[a*Cos[x]^4]*Sin[x])
/24 + (a*Cos[x]^3*Sqrt[a*Cos[x]^4]*Sin[x])/6 + (5*a*Sqrt[a*Cos[x]^4]*Tan[x]
)/16
```

Rubi [A] time = 0.0319987, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$\frac{1}{6}a\sin(x)\cos^3(x)\sqrt{a\cos^4(x)}+\frac{5}{24}a\sin(x)\cos(x)\sqrt{a\cos^4(x)}+\frac{5}{16}a\tan(x)\sqrt{a\cos^4(x)}+\frac{5}{16}ax\sec^2(x)\sqrt{a\cos^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^4)^(3/2),x]
```

```
[Out] (5*a*x*Sqrt[a*Cos[x]^4]*Sec[x]^2)/16 + (5*a*Cos[x]*Sqrt[a*Cos[x]^4]*Sin[x])
/24 + (a*Cos[x]^3*Sqrt[a*Cos[x]^4]*Sin[x])/6 + (5*a*Sqrt[a*Cos[x]^4]*Tan[x]
)/16
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a\cos^4(x)\right)^{3/2} dx = \left(a\sqrt{a\cos^4(x)}\sec^2(x)\right)\int \cos^6(x)\,dx$$
$$= \frac{1}{6}a\cos^3(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{6}\left(5a\sqrt{a\cos^4(x)}\sec^2(x)\right)\int \cos^4(x)\,dx$$
$$= \frac{5}{24}a\cos(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{6}a\cos^3(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{8}\left(5a\sqrt{a\cos^4(x)}\sec^2(x)\right)\int \cos^2(x$$
$$= \frac{5}{24}a\cos(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{6}a\cos^3(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{5}{16}a\sqrt{a\cos^4(x)}\tan(x) + \frac{1}{16}\left(5a\sqrt{a\right.$$
$$= \frac{5}{16}ax\sqrt{a\cos^4(x)}\sec^2(x) + \frac{5}{24}a\cos(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{6}a\cos^3(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{5}{16}a\sqrt{a\,c}$$

Mathematica [A] time = 0.0685168, size = 38, normalized size = 0.49

$$\frac{1}{192}(60x + 45\sin(2x) + 9\sin(4x) + \sin(6x))\sec^6(x)\left(a\cos^4(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^4)^(3/2),x]
```

```
[Out] ((a*Cos[x]^4)^(3/2)*Sec[x]^6*(60*x + 45*Sin[2*x] + 9*Sin[4*x] + Sin[6*x]))/
192
```

Maple [A] time = 0.162, size = 41, normalized size = 0.5

$$\frac{8\sin(x)(\cos(x))^5 + 10(\cos(x))^3\sin(x) + 15\cos(x)\sin(x) + 15x}{48(\cos(x))^6}\left(a(\cos(x))^4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^4)^(3/2),x)
```

```
[Out] 1/48*(a*cos(x)^4)^(3/2)*(8*sin(x)*cos(x)^5+10*cos(x)^3*sin(x)+15*cos(x)*sin
(x)+15*x)/cos(x)^6
```

Maxima [A] time = 2.31156, size = 74, normalized size = 0.95

$$\frac{5}{16}a^{\frac{3}{2}}x + \frac{15a^{\frac{3}{2}}\tan(x)^5 + 40a^{\frac{3}{2}}\tan(x)^3 + 33a^{\frac{3}{2}}\tan(x)}{48\left(\tan(x)^6 + 3\tan(x)^4 + 3\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^(3/2)*x + 1/48*(15*a^(3/2)*tan(x)^5 + 40*a^(3/2)*tan(x)^3 + 33*a^(3/
2)*tan(x))/(tan(x)^6 + 3*tan(x)^4 + 3*tan(x)^2 + 1)
```

Fricas [A] time = 1.08133, size = 128, normalized size = 1.64

$$\frac{\sqrt{a\cos(x)^4}\left(15\,ax + \left(8\,a\cos(x)^5 + 10\,a\cos(x)^3 + 15\,a\cos(x)\right)\sin(x)\right)}{48\,\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(a*cos(x)^4)*(15*a*x + (8*a*cos(x)^5 + 10*a*cos(x)^3 + 15*a*cos(x)
)*sin(x))/cos(x)^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**4)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42684, size = 34, normalized size = 0.44

$$\frac{1}{192}\,a^{\frac{3}{2}}(60\,x + \sin(6\,x) + 9\,\sin(4\,x) + 45\,\sin(2\,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*a^(3/2)*(60*x + sin(6*x) + 9*sin(4*x) + 45*sin(2*x))
```

3.53 $\int \sqrt{a\cos^4(x)}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}\tan(x)\sqrt{a\cos^4(x)}+\frac{1}{2}x\sec^2(x)\sqrt{a\cos^4(x)}$$

```
[Out] (x*Sqrt[a*Cos[x]^4]*Sec[x]^2)/2 + (Sqrt[a*Cos[x]^4]*Tan[x])/2
```

Rubi [A] time = 0.0153305, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$\frac{1}{2}\tan(x)\sqrt{a\cos^4(x)}+\frac{1}{2}x\sec^2(x)\sqrt{a\cos^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[x]^4],x]
```

```
[Out] (x*Sqrt[a*Cos[x]^4]*Sec[x]^2)/2 + (Sqrt[a*Cos[x]^4]*Tan[x])/2
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\cos^4(x)}\,dx &= \left(\sqrt{a\cos^4(x)}\sec^2(x)\right)\int \cos^2(x)\,dx\\ &= \frac{1}{2}\sqrt{a\cos^4(x)}\tan(x)+\frac{1}{2}\left(\sqrt{a\cos^4(x)}\sec^2(x)\right)\int 1\,dx\\ &= \frac{1}{2}x\sqrt{a\cos^4(x)}\sec^2(x)+\frac{1}{2}\sqrt{a\cos^4(x)}\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0137499, size = 25, normalized size = 0.69

$$\frac{1}{2}\sec^2(x)\sqrt{a\cos^4(x)}(x+\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[x]^4],x]
```

```
[Out] (Sqrt[a*Cos[x]^4]*Sec[x]^2*(x + Cos[x]*Sin[x]))/2
```

Maple [A] time = 0.192, size = 22, normalized size = 0.6

$$\frac{\cos(x)\sin(x)+x}{2(\cos(x))^2}\sqrt{a(\cos(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^4)^(1/2),x)
```

```
[Out] 1/2*(a*cos(x)^4)^(1/2)*(cos(x)*sin(x)+x)/cos(x)^2
```

Maxima [A] time = 2.4277, size = 30, normalized size = 0.83

$$\frac{1}{2}\sqrt{a}x+\frac{\sqrt{a}\tan(x)}{2\left(\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*x + 1/2*sqrt(a)*tan(x)/(tan(x)^2 + 1)
```

Fricas [A] time = 1.04887, size = 69, normalized size = 1.92

$$\frac{\sqrt{a\cos(x)^4}(\cos(x)\sin(x)+x)}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a*cos(x)^4)*(cos(x)*sin(x) + x)/cos(x)^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.55724, size = 18, normalized size = 0.5

$$\frac{1}{4}\sqrt{a}(2x + \sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a)*(2*x + sin(2*x))
```

3.54 $\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)\cos(x)}{\sqrt{a\cos^4(x)}}$$

```
[Out] (Cos[x]*Sin[x])/Sqrt[a*Cos[x]^4]
```

Rubi [A] time = 0.0140777, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 3767, 8}

$$\frac{\sin(x)\cos(x)}{\sqrt{a\cos^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[x]^4],x]
```

```
[Out] (Cos[x]*Sin[x])/Sqrt[a*Cos[x]^4]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx &= \frac{\cos^2(x)\int \sec^2(x)\,dx}{\sqrt{a\cos^4(x)}}\\ &= -\frac{\cos^2(x)\,\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)}{\sqrt{a\cos^4(x)}}\\ &= \frac{\cos(x)\sin(x)}{\sqrt{a\cos^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.0055221, size = 15, normalized size = 1.

$$\frac{\sin(x)\cos(x)}{\sqrt{a\cos^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[x]^4],x]
```

```
[Out] (Cos[x]*Sin[x])/Sqrt[a*Cos[x]^4]
```

Maple [A] time = 0.181, size = 14, normalized size = 0.9

$$\cos(x)\sin(x)\frac{1}{\sqrt{a(\cos(x))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^4)^(1/2),x)
```

```
[Out] cos(x)*sin(x)/(a*cos(x)^4)^(1/2)
```

Maxima [A] time = 2.05925, size = 8, normalized size = 0.53

$$\frac{\tan(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] tan(x)/sqrt(a)
```

Fricas [A] time = 1.02808, size = 51, normalized size = 3.4

$$\frac{\sqrt{a\cos(x)^4}\sin(x)}{a\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(x)^4)*sin(x)/(a*cos(x)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.55 $\int \frac{1}{\left(a\cos^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\sin(x)\cos(x)}{a\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^3(x)}{5a\sqrt{a\cos^4(x)}}+\frac{2\sin^2(x)\tan(x)}{3a\sqrt{a\cos^4(x)}}$$

```
[Out] (Cos[x]*Sin[x])/(a*Sqrt[a*Cos[x]^4]) + (2*Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Cos[
x]^4]) + (Sin[x]^2*Tan[x]^3)/(5*a*Sqrt[a*Cos[x]^4])
```

Rubi [A] time = 0.0214227, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3767}

$$\frac{\sin(x)\cos(x)}{a\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^3(x)}{5a\sqrt{a\cos^4(x)}}+\frac{2\sin^2(x)\tan(x)}{3a\sqrt{a\cos^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^4)^(-3/2),x]
```

```
[Out] (Cos[x]*Sin[x])/(a*Sqrt[a*Cos[x]^4]) + (2*Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Cos[
x]^4]) + (Sin[x]^2*Tan[x]^3)/(5*a*Sqrt[a*Cos[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\cos^4(x)\right)^{3/2}}\,dx &= \frac{\cos^2(x)\int \sec^6(x)\,dx}{a\sqrt{a\cos^4(x)}}\\ &= -\frac{\cos^2(x)\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(x)\right)}{a\sqrt{a\cos^4(x)}}\\ &= \frac{\cos(x)\sin(x)}{a\sqrt{a\cos^4(x)}}+\frac{2\sin^2(x)\tan(x)}{3a\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^3(x)}{5a\sqrt{a\cos^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.0275874, size = 30, normalized size = 0.45

$$\frac{\sin(x)\cos(x)(6\cos(2x)+\cos(4x)+8)}{15\left(a\cos^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^4)^(-3/2),x]
```

```
[Out] (Cos[x]*(8 + 6*Cos[2*x] + Cos[4*x])*Sin[x])/(15*(a*Cos[x]^4)^(3/2))
```

Maple [A] time = 0.114, size = 29, normalized size = 0.4

$$\frac{\sin(x)\left(8(\cos(x))^4+4(\cos(x))^2+3\right)\cos(x)}{15}\left(a(\cos(x))^4\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^4)^(3/2),x)
```

```
[Out] 1/15*sin(x)*(8*cos(x)^4+4*cos(x)^2+3)*cos(x)/(a*cos(x)^4)^(3/2)
```

Maxima [A] time = 1.8994, size = 30, normalized size = 0.45

$$\frac{3\tan(x)^5+10\tan(x)^3+15\tan(x)}{15a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^(3/2)
```

Fricas [A] time = 1.04243, size = 101, normalized size = 1.51

$$\frac{\sqrt{a\cos(x)^4}\left(8\cos(x)^4+4\cos(x)^2+3\right)\sin(x)}{15a^2\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*sqrt(a*cos(x)^4)*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^2*cos(x)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**4)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.56 $\int \frac{1}{\left(a\cos^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{\sin(x)\cos(x)}{a^2\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^7(x)}{9a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan^5(x)}{7a^2\sqrt{a\cos^4(x)}}+\frac{6\sin^2(x)\tan^3(x)}{5a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan(x)}{3a^2\sqrt{a\cos^4(x)}}$$

```
[Out] (Cos[x]*Sin[x])/(a^2*Sqrt[a*Cos[x]^4]) + (4*Sin[x]^2*Tan[x])/(3*a^2*Sqrt[a*
Cos[x]^4]) + (6*Sin[x]^2*Tan[x]^3)/(5*a^2*Sqrt[a*Cos[x]^4]) + (4*Sin[x]^2*T
an[x]^5)/(7*a^2*Sqrt[a*Cos[x]^4]) + (Sin[x]^2*Tan[x]^7)/(9*a^2*Sqrt[a*Cos[x
]^4])
```

Rubi [A] time = 0.0321276, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3767}

$$\frac{\sin(x)\cos(x)}{a^2\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^7(x)}{9a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan^5(x)}{7a^2\sqrt{a\cos^4(x)}}+\frac{6\sin^2(x)\tan^3(x)}{5a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan(x)}{3a^2\sqrt{a\cos^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^4)^(-5/2),x]
```

```
[Out] (Cos[x]*Sin[x])/(a^2*Sqrt[a*Cos[x]^4]) + (4*Sin[x]^2*Tan[x])/(3*a^2*Sqrt[a*
Cos[x]^4]) + (6*Sin[x]^2*Tan[x]^3)/(5*a^2*Sqrt[a*Cos[x]^4]) + (4*Sin[x]^2*T
an[x]^5)/(7*a^2*Sqrt[a*Cos[x]^4]) + (Sin[x]^2*Tan[x]^7)/(9*a^2*Sqrt[a*Cos[x
]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\cos^4(x)\right)^{5/2}}\,dx &= \frac{\cos^2(x)\int \sec^{10}(x)\,dx}{a^2\sqrt{a\cos^4(x)}}\\ &= -\frac{\cos^2(x)\,\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx,x,-\tan(x)\right)}{a^2\sqrt{a\cos^4(x)}}\\ &= \frac{\cos(x)\sin(x)}{a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan(x)}{3a^2\sqrt{a\cos^4(x)}}+\frac{6\sin^2(x)\tan^3(x)}{5a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan^5(x)}{7a^2\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^7(x)}{9a^2\sqrt{a\cos^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.051605, size = 47, normalized size = 0.4

$$\frac{(130\cos(2x)+46\cos(4x)+10\cos(6x)+\cos(8x)+128)\tan(x)\sec^6(x)}{315a^2\sqrt{a\cos^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^4)^(-5/2),x]
```

```
[Out] ((128 + 130*Cos[2*x] + 46*Cos[4*x] + 10*Cos[6*x] + Cos[8*x])*Sec[x]^6*Tan[x
])/(315*a^2*Sqrt[a*Cos[x]^4])
```

Maple [A] time = 0.165, size = 41, normalized size = 0.4

$$\frac{\sin(x)\left(128\,(\cos(x))^8+64\,(\cos(x))^6+48\,(\cos(x))^4+40\,(\cos(x))^2+35\right)\cos(x)}{315}\left(a\,(\cos(x))^4\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^4)^(5/2),x)
```

```
[Out] 1/315*sin(x)*(128*cos(x)^8+64*cos(x)^6+48*cos(x)^4+40*cos(x)^2+35)*cos(x)/(
a*cos(x)^4)^(5/2)
```

Maxima [A] time = 1.88253, size = 46, normalized size = 0.39

$$\frac{35\tan(x)^9+180\tan(x)^7+378\tan(x)^5+420\tan(x)^3+315\tan(x)}{315\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(x
))/a^(5/2)
```

Fricas [A] time = 1.08485, size = 147, normalized size = 1.26

$$\frac{\left(128\cos(x)^8+64\cos(x)^6+48\cos(x)^4+40\cos(x)^2+35\right)\sqrt{a\cos(x)^4}\sin(x)}{315\,a^3\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(128*cos(x)^8 + 64*cos(x)^6 + 48*cos(x)^4 + 40*cos(x)^2 + 35)*sqrt(a*
cos(x)^4)*sin(x)/(a^3*cos(x)^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30331, size = 46, normalized size = 0.39

$$\frac{35 \tan(x)^9 + 180 \tan(x)^7 + 378 \tan(x)^5 + 420 \tan(x)^3 + 315 \tan(x)}{315 a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(x
))/a^(5/2)
```

3.57 $\int (b\cos^m(c+dx))^n\,dx$

Optimal. Leaf size=78

$$-\frac{\sin(c+dx)\cos(c+dx)\,(b\cos^m(c+dx))^n\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\cos^2(c+dx)\right)}{d(mn+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x]^m)^n*Hypergeometric2F1[1/2, (1 + m*n)/2, (3
 + m*n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m*n)*Sqrt[Sin[c + d*x]^2])
)
```

Rubi [A] time = 0.0347181, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3208, 2643}

$$-\frac{\sin(c+dx)\cos(c+dx)\,(b\cos^m(c+dx))^n\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\cos^2(c+dx)\right)}{d(mn+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x]^m)^n,x]
```

```
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x]^m)^n*Hypergeometric2F1[1/2, (1 + m*n)/2, (3
 + m*n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m*n)*Sqrt[Sin[c + d*x]^2])
)
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos^m(c+dx))^n\,dx &= \left(\cos^{-mn}(c+dx)\,(b\cos^m(c+dx))^n\right)\int \cos^{mn}(c+dx)\,dx\\ &= -\frac{\cos(c+dx)\,(b\cos^m(c+dx))^n\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+mn);\frac{1}{2}(3+mn);\cos^2(c+dx)\right)\sin(c+dx)}{d(1+mn)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.061062, size = 72, normalized size = 0.92

$$-\frac{\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(b\cos^m(c+dx)\right)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\cos^2(c+dx)\right)}{d(mn+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x]^m)^n,x]
```

```
[Out] -(((b*Cos[c + d*x]^m)^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (1 + m*n)/2, (3
 + m*n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1 + m*n)))
```

Maple [F] time = 0.274, size = 0, normalized size = 0.

$$\int \left(b\,(\cos\,(dx+c))^m\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c)^m)^n,x)
```

```
[Out] int((b*cos(d*x+c)^m)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\,(dx+c)^m\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c)^m)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)^m)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\,(dx+c)^m\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c)^m)^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^m)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos^m\,(c+dx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c)**m)**n,x)
```

```
[Out] Integral((b*cos(c + d*x)**m)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \cos \left(dx + c\right)^{m}\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c)^m)^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)^m)^n, x)
```

3.58 $\int (c\cos^m(a+bx))^{5/2}\,dx$

Optimal. Leaf size=89

$$-\frac{2c^2\sin(a+bx)\cos^{2m+1}(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\cos^2(a+bx)\right)}{b(5m+2)\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-2*c^2*Cos[a + b*x]^(1 + 2*m)*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/2
, (2 + 5*m)/4, (6 + 5*m)/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + 5*m)*Sqrt
[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0400767, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$-\frac{2c^2\sin(a+bx)\cos^{2m+1}(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\cos^2(a+bx)\right)}{b(5m+2)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x]^m)^(5/2),x]
```

```
[Out] (-2*c^2*Cos[a + b*x]^(1 + 2*m)*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/2
, (2 + 5*m)/4, (6 + 5*m)/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + 5*m)*Sqrt
[Sin[a + b*x]^2])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos^m(a+bx))^{5/2}\,dx = \left(c^2\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\right)\int \cos^{\frac{5m}{2}}(a+bx)\,dx$$
$$= -\frac{2c^2\cos^{1+2m}(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+5m);\frac{1}{4}(6+5m);\cos^2(a+bx)\right)\sin(a+}{b(2+5m)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.172526, size = 74, normalized size = 0.83

$$-\frac{2\sqrt{\sin^2(a+bx)}\cot(a+bx)\,(c\cos^m(a+bx))^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\cos^2(a+bx)\right)}{b(5m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x]^m)^(5/2),x]
```

```
[Out] (-2*(c*Cos[a + b*x]^m)^(5/2)*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 + 5*m)/
4, (6 + 5*m)/4, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(2 + 5*m))
```

Maple [F] time = 0.419, size = 0, normalized size = 0.

$$\int \left(c\left(\cos\left(bx+a\right)\right)^{m}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a)^m)^(5/2),x)
```

```
[Out] int((c*cos(b*x+a)^m)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\cos\left(bx+a\right)^{m}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)**m)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\cos \left(bx+a\right)^{m}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(5/2), x)
```

3.59 $\int (c\cos^m(a+bx))^{3/2}\,dx$

Optimal. Leaf size=83

$$-\frac{2c\sin(a+bx)\cos^{m+1}(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\cos^2(a+bx)\right)}{b(3m+2)\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-2*c*Cos[a + b*x]^(1 + m)*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/2, (2
 + 3*m)/4, (3*(2 + m))/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + 3*m)*Sqrt[S
in[a + b*x]^2])
```

Rubi [A] time = 0.039055, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$-\frac{2c\sin(a+bx)\cos^{m+1}(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\cos^2(a+bx)\right)}{b(3m+2)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x]^m)^(3/2),x]
```

```
[Out] (-2*c*Cos[a + b*x]^(1 + m)*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/2, (2
 + 3*m)/4, (3*(2 + m))/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + 3*m)*Sqrt[S
in[a + b*x]^2])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (c\cos^m(a+bx))^{3/2}\,dx &= \left(c\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\right)\int \cos^{\frac{3m}{2}}(a+bx)\,dx\\ &= -\frac{2c\cos^{1+m}(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+3m);\frac{3(2+m)}{4};\cos^2(a+bx)\right)\sin(a+bx)}{b(2+3m)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.118886, size = 72, normalized size = 0.87

$$-\frac{2\sqrt{\sin^2(a+bx)}\cot(a+bx)\left(c\cos^m(a+bx)\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\cos^2(a+bx)\right)}{b(3m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x]^m)^(3/2),x]
```

```
[Out] (-2*(c*Cos[a + b*x]^m)^(3/2)*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 + 3*m)/
4, (3*(2 + m))/4, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(2 + 3*m))
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int \left(c\left(\cos\left(bx+a\right)\right)^{m}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a)^m)^(3/2),x)
```

```
[Out] int((c*cos(b*x+a)^m)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\cos\left(bx+a\right)^{m}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x + a)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\cos^{m}\left(a+bx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)**m)**(3/2),x)

[Out] Integral((c*cos(a + b*x)**m)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\cos\left(bx+a\right)^{m}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(3/2),x, algorithm="giac")

[Out] integrate((c*cos(b*x + a)^m)^(3/2), x)
```

3.60 $\int \sqrt{c\cos^m(a+bx)}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sin(a+bx)\cos(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\cos^2(a+bx)\right)}{b(m+2)\sqrt{\sin^2(a+bx)}}$$

```
[Out] (-2*Cos[a + b*x]*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/2, (2 + m)/4, (
6 + m)/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + m)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0392015, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$-\frac{2\sin(a+bx)\cos(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\cos^2(a+bx)\right)}{b(m+2)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Cos[a + b*x]^m],x]
```

```
[Out] (-2*Cos[a + b*x]*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/2, (2 + m)/4, (
6 + m)/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + m)*Sqrt[Sin[a + b*x]^2])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c\cos^m(a+bx)}\,dx &= \left(\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\right)\int \cos^{\frac{m}{2}}(a+bx)\,dx\\ &= -\frac{2\cos(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{4};\frac{6+m}{4};\cos^2(a+bx)\right)\sin(a+bx)}{b(2+m)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0704345, size = 68, normalized size = 0.92

$$-\frac{2\sqrt{\sin^2(a+bx)}\cot(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\cos^2(a+bx)\right)}{b(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Cos[a + b*x]^m],x]
```

```
[Out] (-2*Sqrt[c*Cos[a + b*x]^m]*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 + m)/4, (
6 + m)/4, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(2 + m))
```

Maple [F] time = 0.239, size = 0, normalized size = 0.

$$\int \sqrt{c (\cos (bx + a))^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a)^m)^(1/2),x)
```

```
[Out] int((c*cos(b*x+a)^m)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \cos (bx + a)^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*cos(b*x + a)^m), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \cos^m (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)**m)**(1/2),x)
```

```
[Out] Integral(sqrt(c*cos(a + b*x)**m), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \cos(bx+a)^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*cos(b*x + a)^m), x)
```

3.61 $\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sin(a+bx)\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\cos^2(a+bx)\right)}{b(2-m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

```
[Out] (-2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Cos[a + b*x]^
2]*Sin[a + b*x])/(b*(2 - m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0466379, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$-\frac{2\sin(a+bx)\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\cos^2(a+bx)\right)}{b(2-m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Cos[a + b*x]^m],x]
```

```
[Out] (-2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Cos[a + b*x]^
2]*Sin[a + b*x])/(b*(2 - m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt[Sin[a + b*x]^2])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx &= \frac{\cos^{\frac{m}{2}}(a+bx)\int \cos^{-\frac{m}{2}}(a+bx)\,dx}{\sqrt{c\cos^m(a+bx)}}\\ &= -\frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\cos^2(a+bx)\right)\sin(a+bx)}{b(2-m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0749802, size = 72, normalized size = 0.9

$$\frac{2\sqrt{\sin^2(a+bx)}\cot(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\cos^2(a+bx)\right)}{b(m-2)\sqrt{c\cos^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Cos[a + b*x]^m],x]
```

```
[Out] (2*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Cos[a + b*x]^2
]*Sqrt[Sin[a + b*x]^2])/(b*(-2 + m)*Sqrt[c*Cos[a + b*x]^m])
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\left(\cos\left(bx+a\right)\right)^{m}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a)^m)^(1/2),x)
```

```
[Out] int(1/(c*cos(b*x+a)^m)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\cos\left(bx+a\right)^{m}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*cos(b*x + a)^m), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\cos^{m}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)**m)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*cos(a + b*x)**m), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\cos(bx+a)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*cos(b*x + a)^m), x)
```

3.62 $\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\sin(a+bx)\cos^{1-m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\cos^2(a+bx)\right)}{bc(2-3m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

```
[Out] (-2*Cos[a + b*x]^(1 - m)*Hypergeometric2F1[1/2, (2 - 3*m)/4, (3*(2 - m))/4,
 Cos[a + b*x]^2]*Sin[a + b*x])/(b*c*(2 - 3*m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt[S
in[a + b*x]^2])
```

Rubi [A] time = 0.0465239, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$-\frac{2\sin(a+bx)\cos^{1-m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\cos^2(a+bx)\right)}{bc(2-3m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x]^m)^(-3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^(1 - m)*Hypergeometric2F1[1/2, (2 - 3*m)/4, (3*(2 - m))/4,
 Cos[a + b*x]^2]*Sin[a + b*x])/(b*c*(2 - 3*m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt[S
in[a + b*x]^2])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx &= \frac{\cos^{\frac{m}{2}}(a+bx)\int \cos^{-\frac{3m}{2}}(a+bx)\,dx}{c\sqrt{c\cos^m(a+bx)}}\\ &= -\frac{2\cos^{1-m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\cos^2(a+bx)\right)\sin(a+bx)}{bc(2-3m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.106103, size = 72, normalized size = 0.81

$$-\frac{\sqrt{\sin^2(a+bx)}\cot(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);-\frac{3}{4}(m-2);\cos^2(a+bx)\right)}{\left(b-\frac{3bm}{2}\right)(c\cos^m(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x]^m)^(-3/2),x]
```

```
[Out] -((Cot[a + b*x]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (-3*(-2 + m))/4, Cos[a
+ b*x]^2]*Sqrt[Sin[a + b*x]^2])/((b - (3*b*m)/2)*(c*Cos[a + b*x]^m)^(3/2)))
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int \left(c\left(\cos(bx+a)\right)^{m}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a)^m)^(3/2),x)
```

```
[Out] int(1/(c*cos(b*x+a)^m)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\cos(bx+a)^{m}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\cos^{m}(a+bx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)**m)**(3/2),x)
```

```
[Out] Integral((c*cos(a + b*x)**m)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\cos\left(bx+a\right)^{m}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(-3/2), x)
```

3.63 $\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\sin(a+bx)\cos^{1-2m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\cos^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

```
[Out] (-2*Cos[a + b*x]^(1 - 2*m)*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4,
 Cos[a + b*x]^2]*Sin[a + b*x])/(b*c^2*(2 - 5*m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt
[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0453095, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$-\frac{2\sin(a+bx)\cos^{1-2m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\cos^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x]^m)^(-5/2),x]
```

```
[Out] (-2*Cos[a + b*x]^(1 - 2*m)*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4,
 Cos[a + b*x]^2]*Sin[a + b*x])/(b*c^2*(2 - 5*m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt
[Sin[a + b*x]^2])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx &= \frac{\cos^{\frac{m}{2}}(a+bx)\int \cos^{-\frac{5m}{2}}(a+bx)\,dx}{c^2\sqrt{c\cos^m(a+bx)}}\\ &= -\frac{2\cos^{1-2m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\cos^2(a+bx)\right)\sin(a+bx)}{bc^2(2-5m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.106179, size = 74, normalized size = 0.83

$$-\frac{\sqrt{\sin^2(a+bx)}\cot(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\cos^2(a+bx)\right)}{\left(b-\frac{5bm}{2}\right)(c\cos^m(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x]^m)^(-5/2),x]
```

```
[Out] -((Cot[a + b*x]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Cos[a + b*
x]^2]*Sqrt[Sin[a + b*x]^2])/((b - (5*b*m)/2)*(c*Cos[a + b*x]^m)^(5/2)))
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int \left(c\left(\cos\left(bx+a\right)\right)^{m}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a)^m)^(5/2),x)
```

```
[Out] int(1/(c*cos(b*x+a)^m)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\cos\left(bx+a\right)^{m}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)**m)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\cos\left(bx+a\right)^{m}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(-5/2), x)
```

3.64 $\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx$

Optimal. Leaf size=24

$$\frac{\tan(a+bx)\,(c\cos^m(a+bx))^{\frac{1}{m}}}{b}$$

```
[Out] ((c*Cos[a + b*x]^m)^m^(-1)*Tan[a + b*x])/b
```

Rubi [A] time = 0.0198843, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2637}

$$\frac{\tan(a+bx)\,(c\cos^m(a+bx))^{\frac{1}{m}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x]^m)^m^(-1),x]
```

```
[Out] ((c*Cos[a + b*x]^m)^m^(-1)*Tan[a + b*x])/b
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx &= \left((c\cos^m(a+bx))^{\frac{1}{m}}\sec(a+bx)\right)\int \cos(a+bx)\,dx\\ &= \frac{(c\cos^m(a+bx))^{\frac{1}{m}}\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.029611, size = 24, normalized size = 1.

$$\frac{\tan(a+bx)\,(c\cos^m(a+bx))^{\frac{1}{m}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x]^m)^m^(-1),x]
```

```
[Out] ((c*Cos[a + b*x]^m)^m^(-1)*Tan[a + b*x])/b
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int \sqrt[m]{c\left(\cos\left(bx+a\right)\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a)^m)^(1/m),x)
```

```
[Out] int((c*cos(b*x+a)^m)^(1/m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\cos\left(bx+a\right)^{m}\right)^{\left(\frac{1}{m}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/m),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(1/m), x)
```

Fricas [A] time = 1.0493, size = 32, normalized size = 1.33

$$\frac{c^{\left(\frac{1}{m}\right)}\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/m),x, algorithm="fricas")
```

```
[Out] c^(1/m)*sin(b*x + a)/b
```

Sympy [A] time = 1.54197, size = 65, normalized size = 2.71

$$\begin{cases} x\left(c\cos^{m}\left(a\right)\right)^{\frac{1}{m}} & \text{for}\: b=0 \\ x\left(0^{m}c\right)^{\frac{1}{m}} & \text{for}\: a=-bx+\frac{\pi}{2} \vee a=-bx+\frac{3\pi}{2} \\ \frac{c^{\frac{1}{m}}\left(\cos^{m}\left(a+bx\right)\right)^{\frac{1}{m}}\sin\left(a+bx\right)}{b\cos\left(a+bx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)**m)**(1/m),x)
```

```
[Out] Piecewise((x*(c*cos(a)**m)**(1/m), Eq(b, 0)), (x*(0**m*c)**(1/m), Eq(a, -b*
x + pi/2) | Eq(a, -b*x + 3*pi/2)), (c**(1/m)*(cos(a + b*x)**m)**(1/m)*sin(a
```

```
+ b*x)/(b*cos(a + b*x)), True))
```

Giac [B] time = 8.07252, size = 405, normalized size = 16.88

$$\frac{2\left(|c|^{\left(\frac{1}{m}\right)}\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{\pi\mathrm{sgn}(c)}{4\,m}-\frac{\pi}{4\,m}\right)^{2}\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}-|c|^{\left(\frac{1}{m}\right)}\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{\pi\mathrm{sgn}(c)}{4\,m}-\frac{\pi}{4\,m}\right)^{2}\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}{b\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{\pi\mathrm{sgn}(c)}{4\,m}-\frac{\pi}{4\,m}\right)^{2}\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}+2\,b\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{\pi\mathrm{sgn}(c)}{4\,m}-\frac{\pi}{4\,m}\right)^{2}\tan\left(\frac{1}{2}\,bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/m),x, algorithm="giac")
```

```
[Out] 2*(abs(c)^(1/m)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/m - 1/4*pi/m)^2*tan(1/2
*b*x + 1/2*a)^3 - abs(c)^(1/m)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/m - 1/4*
pi/m)^2*tan(1/2*b*x + 1/2*a) + 4*abs(c)^(1/m)*tan(1/2*b*x + 1/2*a + 1/4*pi*
sgn(c)/m - 1/4*pi/m)*tan(1/2*b*x + 1/2*a)^2 - abs(c)^(1/m)*tan(1/2*b*x + 1/
2*a)^3 + abs(c)^(1/m)*tan(1/2*b*x + 1/2*a))/(b*tan(1/2*b*x + 1/2*a + 1/4*pi
*sgn(c)/m - 1/4*pi/m)^2*tan(1/2*b*x + 1/2*a)^4 + 2*b*tan(1/2*b*x + 1/2*a +
1/4*pi*sgn(c)/m - 1/4*pi/m)^2*tan(1/2*b*x + 1/2*a)^2 + b*tan(1/2*b*x + 1/2*
a)^4 + b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/m - 1/4*pi/m)^2 + 2*b*tan(1/2*
b*x + 1/2*a)^2 + b)
```

3.65 $\int (a(b\cos(c+dx))^p)^n\,dx$

Optimal. Leaf size=80

$$-\frac{\sin(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\cos^2(c+dx)\right)(a(b\cos(c+dx))^p)^n}{d(np+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((Cos[c + d*x]*(a*(b*Cos[c + d*x])^p)^n*Hypergeometric2F1[1/2, (1 + n*p)/2
, (3 + n*p)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + n*p)*Sqrt[Sin[c + d*x]
^2]))
```

Rubi [A] time = 0.0461189, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$-\frac{\sin(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\cos^2(c+dx)\right)(a(b\cos(c+dx))^p)^n}{d(np+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*Cos[c + d*x])^p)^n,x]
```

```
[Out] -((Cos[c + d*x]*(a*(b*Cos[c + d*x])^p)^n*Hypergeometric2F1[1/2, (1 + n*p)/2
, (3 + n*p)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + n*p)*Sqrt[Sin[c + d*x]
^2]))
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a(b\cos(c+dx))^p)^n\,dx &= \left((b\cos(c+dx))^{-np}(a(b\cos(c+dx))^p)^n\right)\int (b\cos(c+dx))^{np}\,dx\\ &= -\frac{\cos(c+dx)\,(a(b\cos(c+dx))^p)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\cos^2(c+dx)\right)\sin(c+dx)}{d(1+np)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0566544, size = 74, normalized size = 0.92

$$-\frac{\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\cos^2(c+dx)\right)(a(b\cos(c+dx))^p)^n}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*Cos[c + d*x])^p)^n,x]
```

```
[Out] -(((a*(b*Cos[c + d*x])^p)^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2
, (3 + n*p)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1 + n*p)))
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int \left(a\,(b\cos(dx+c))^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*cos(d*x+c))^p)^n,x)
```

```
[Out] int((a*(b*cos(d*x+c))^p)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b\cos(dx+c))^p\,a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cos(d*x+c))^p)^n,x, algorithm="maxima")
```

```
[Out] integrate(((b*cos(d*x + c))^p*a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left((b\cos(dx+c))^p\,a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cos(d*x+c))^p)^n,x, algorithm="fricas")
```

```
[Out] integral(((b*cos(d*x + c))^p*a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\,(b\cos(c+dx))^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cos(d*x+c))**p)**n,x)
```

```
[Out] Integral((a*(b*cos(c + d*x))**p)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b\cos(dx+c))^p a\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cos(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*cos(d*x + c))^p*a)^n, x)
```

3.66 $\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=123

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^4d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^2d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77d}+\frac{30b\sqrt{\cos(c+dx)}\ldots}{77d\sqrt{b\cos\ldots}}$$

```
[Out] (30*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c + d*
x]]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*d) + (18*(b*Cos[c + d*x])
^(5/2)*Sin[c + d*x])/(77*b^2*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(
11*b^4*d)
```

Rubi [A] time = 0.0900455, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^4d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^2d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77d}+\frac{30b\sqrt{\cos(c+dx)}\ldots}{77d\sqrt{b\cos\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (30*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c + d*
x]]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*d) + (18*(b*Cos[c + d*x])
^(5/2)*Sin[c + d*x])/(77*b^2*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(
11*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^5}$$
$$= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^4d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b^3}$$
$$= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^2d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^4d} + \frac{45\int (b\cos(c+d}{77b}$$
$$= \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^2d} + \frac{2(b\cos(c+dx))^{9/}}{11b^4d}$$
$$= \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^2d} + \frac{2(b\cos(c+dx))^{9/}}{11b^4d}$$
$$= \frac{30b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx)}{77b}$$

Mathematica [A] time = 0.219839, size = 83, normalized size = 0.67

$$\frac{\sqrt{b\cos(c+dx)}\left(240F\left(\frac{1}{2}(c+dx)\middle|2\right)+(290\sin(c+dx)+57\sin(3(c+dx))+7\sin(5(c+dx)))\sqrt{\cos(c+dx)}\right)}{616d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(240*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(
290*Sin[c + d*x] + 57*Sin[3*(c + d*x)] + 7*Sin[5*(c + d*x)])))/(616*d*Sqrt[
Cos[c + d*x]])
```

Maple [A] time = 1.885, size = 234, normalized size = 1.9

$$-\frac{2b}{77d}\sqrt{b\left(2\left(\cos(1/2dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(448\left(\cos(1/2dx+c/2)\right)^{13}-1568\left(\cos(1/2dx+c/2)\right)^{11}+2384\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(448*cos(
1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-2040
*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+15
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)
^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\cos(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^5, x)
```

3.67 $\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^3d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45bd}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]
]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d) + (2*(b*Cos[c + d*x]
)^(7/2)*Sin[c + d*x])/(9*b^3*d)
```

Rubi [A] time = 0.069299, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2640, 2639}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^3d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45bd}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]
]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d) + (2*(b*Cos[c + d*x]
)^(7/2)*Sin[c + d*x])/(9*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^4}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d} + \frac{7\int (b\cos(c+dx))^{5/2}\,dx}{9b^2}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d} + \frac{7}{15}\int \sqrt{b\cos(}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d} + \frac{(7\sqrt{b\cos(c+}}{15}$$
$$= \frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2(b\cos(c}{}$$

Mathematica [A] time = 0.153787, size = 75, normalized size = 0.77

$$\frac{\sqrt{b\cos(c+dx)}\left(168E\left(\frac{1}{2}(c+dx)\middle|2\right)+(38\sin(2(c+dx))+5\sin(4(c+dx)))\sqrt{\cos(c+dx)}\right)}{180d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(168*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(
38*Sin[2*(c + d*x)] + 5*Sin[4*(c + d*x)])))/(180*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 1.934, size = 221, normalized size = 2.3

$$-\frac{2b}{45d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(160\left(\cos(1/2\,dx+c/2)\right)^{11}-480\left(\cos(1/2\,dx+c/2)\right)^9+616\,(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(160*cos(
1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*cos
(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*
cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^4, x)
```

3.68 $\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*
x]]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(7*b^2*d)
```

Rubi [A] time = 0.0658559, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*
x]]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(7*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b^3}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{5\int (b\cos(c+dx))^{3/2}\,dx}{7b}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{1}{21}(5b)\int \frac{1}{\sqrt{b\cos(c}}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{\left(5b\sqrt{\cos(c+dx)}\right)}{21\sqrt{b\cos(c}}$$
$$= \frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))}{7b^2}$$

Mathematica [A] time = 0.145203, size = 73, normalized size = 0.77

$$\frac{\sqrt{b\cos(c+dx)}\left(20F\left(\frac{1}{2}(c+dx)\middle|2\right)+(23\sin(c+dx)+3\sin(3(c+dx)))\sqrt{\cos(c+dx)}\right)}{42d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(2
3*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.051, size = 208, normalized size = 2.2

$$-\frac{2b}{21d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(48\left(\cos(1/2\,dx+c/2)\right)^9-120\left(\cos(1/2\,dx+c/2)\right)^7+128\left(\cos(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(48*cos(1
/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(1/
2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*co
s(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}\cos(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.69 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.0409837, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2640, 2639}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\int (b\cos(c+dx))^{5/2}\,dx}{b^2}$$
$$= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{3}{5}\int \sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}$$
$$= \frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

Mathematica [A] time = 0.0619382, size = 62, normalized size = 0.9

$$\frac{\sqrt{b\cos(c+dx)}\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\sqrt{\cos(c+dx)}\right)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin
[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 1.817, size = 211, normalized size = 3.1

$$-\frac{2b}{5d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+8\left(\sin(1/2\,dx+c/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-8*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*
(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*
(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}\cos(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cos(dx+c)}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

3.70 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0428716, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\,dx}{b}$$
$$= \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}b\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{\left(b\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$
$$= \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.0641147, size = 61, normalized size = 0.91

$$\frac{2(b\cos(c+dx))^{3/2}\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*S
in[c + d*x]))/(3*b*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 1.864, size = 188, normalized size = 2.8

$$-\frac{2b}{3d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

3.71 $\int \sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0200229, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2640, 2639}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.022751, size = 38, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 1.457, size = 142, normalized size = 3.7

$$2\,\frac{\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\,b\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\,\mathrm{EllipticE}\left(\right.}{\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+c/2))^2\right)}\sin(1/2\,dx+c/2)\,\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x)), x)
```

Giac **[F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c)), x)
```

3.72 $\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx$

Optimal. Leaf size=39

$$\frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0292921, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 2642, 2641}

$$\frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx &= b\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{\left(b\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0313062, size = 39, normalized size = 1.

$$\frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 1.5, size = 142, normalized size = 3.6

$$-2\,\frac{\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\,b\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\mathrm{EllipticF}\left(\mathrm{co}\right.}{\sqrt{-b\left(2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}-\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}\right)}\sin\left(1/2\,dx+c/2\right)\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c
)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/sin(
1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos\left(c+dx\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

3.73 $\int \sqrt{b \cos(c+dx)} \sec^2(c+dx) \, dx$

Optimal. Leaf size=63

$$\frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
+ (2*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0546447, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
+ (2*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\int \sqrt{b\cos(c+dx)}\,dx\\ &= \frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0538614, size = 48, normalized size = 0.76

$$\frac{2b\left(\sin(c+dx)-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(d*S
qrt[b*Cos[c + d*x]])
```

Maple [A] time = 1.892, size = 166, normalized size = 2.6

$$-2\,\frac{b\sqrt{-2\,b\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2\,b}\left(\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\right.\right.}{\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+c/2))^2\right)}\sin(1/2\,dx+c/2)\sqrt{b\left(2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*((sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+
1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+
1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cos\left(c+dx\right)}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

3.74 $\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx$

Optimal. Leaf size=70

$$\frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]
]) + (2*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0568187, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2642, 2641}

$$\frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]
]) + (2*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx = b^3\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}b\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left(b\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$
$$= \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0781676, size = 49, normalized size = 0.7

$$\frac{2b\left(\tan(c+dx)+\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] (2*b*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d*Sq
rt[b*Cos[c + d*x]])
```

Maple [B] time = 1.944, size = 239, normalized size = 3.4

$$-\frac{2b}{3d}\left(-2\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\left(\sin(1/2\,dx+c/2)\right)^2+\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*b*(b*(2*cos(1/2*d*x+1/2*c)^
2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*
d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

3.75 $\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx$

Optimal. Leaf size=95

$$\frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]
) + (2*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*b*Sin[c + d*x])/
(5*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0734399, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]
) + (2*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*b*Sin[c + d*x])/
(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx = b^4\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(3b^2\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{3}{5}\int \sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}$$
$$= -\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.233497, size = 69, normalized size = 0.73

$$\frac{2\sec^2(c+dx)\sqrt{b\cos(c+dx)}\left(\frac{3}{2}\sin(2(c+dx)) + \tan(c+dx) - 3\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(-3*Cos[c + d*x]^(3/2)*EllipticE[(c
+ d*x)/2, 2] + (3*Sin[2*(c + d*x)])/2 + Tan[c + d*x]))/(5*d)
```

Maple [B] time = 2.887, size = 363, normalized size = 3.8

$$\frac{2}{5d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+2
4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

3.76 $\int \sqrt{b\cos(c+dx)}\sec^5(c+dx)\,dx$

Optimal. Leaf size=98

$$\frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^2\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b^2*Sin[c +
d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0723142, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2642, 2641}

$$\frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^2\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^5,x]
```

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b^2*Sin[c +
d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{b\cos(c+dx)}\sec^5(c+dx)\,dx &= b^5\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx\\
&= \frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}\left(5b^3\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\
&= \frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^2\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{1}{21}(5b)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^2\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{\left(5b\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}\\
&= \frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^2\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.194016, size = 69, normalized size = 0.7

$$
\frac{\sec^3(c+dx)\sqrt{b\cos(c+dx)}\left(5\sin(2(c+dx))+6\tan(c+dx)+10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{21d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^5,x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(10*Cos[c + d*x]^(5/2)*EllipticF[(c +
d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Maple [B] time = 1.804, size = 396, normalized size = 4.

$$
-\frac{2\,b}{21\,d}\left(-40\,\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,(\sin(1/2\,dx+c/2))^6+6
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-3
0*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c))*b*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \sqrt{b\cos(dx+c)}\sec(dx+c)^5\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

3.77 $\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx$

Optimal. Leaf size=123

$$\frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x
]]) + (2*b^5*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^3*Sin[c + d
*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*b*Sin[c + d*x])/(15*d*Sqrt[b*Cos[c
 + d*x]])
```

Rubi [A] time = 0.0969328, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^6,x]
```

```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x
]]) + (2*b^5*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^3*Sin[c + d
*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*b*Sin[c + d*x])/(15*d*Sqrt[b*Cos[c
 + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx = b^6\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}\left(7b^4\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{1}{15}\left(7b^2\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} - \frac{7}{15}\int \sqrt{b\cos}$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} - \frac{\left(7\sqrt{b\cos(c\right.}{15}$$
$$= -\frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/}}$$

Mathematica [A] time = 0.21092, size = 79, normalized size = 0.64

$$\frac{\sec^5(c+dx)\sqrt{b\cos(c+dx)}\left(150\sin(c+dx)+91\sin(3(c+dx))+21\sin(5(c+dx))-336\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right.\right.}{360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^6,x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^5*(-336*Cos[c + d*x]^(9/2)*EllipticE[(c
 + d*x)/2, 2] + 150*Sin[c + d*x] + 91*Sin[3*(c + d*x)] + 21*Sin[5*(c + d*x)]
))/(360*d)
```

Maple [B] time = 3.457, size = 412, normalized size = 3.4

$$-2\,\frac{\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2b}\,b}{\sin(1/2\,dx+c/2)\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)}d}\left(-\frac{\cos(1/2\,dx+c/2)\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,d\right.}}{144\,b\left((\cos(1/2\,dx+c/2))^2-1/2\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-1/144*cos(
1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1
/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*
d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos(dx+c)}\sec(dx+c)^6\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)
```

3.78 $\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^3d}+\frac{30b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77bd}+\frac{30b\sin(c+\ldots}{}$$

```
[Out] (30*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c +
d*x]]) + (30*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*d) + (18*(b*Cos[c + d
*x])^(5/2)*Sin[c + d*x])/(77*b*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])
/(11*b^3*d)
```

Rubi [A] time = 0.0859398, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^3d}+\frac{30b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77bd}+\frac{30b\sin(c+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (30*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c +
d*x]]) + (30*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*d) + (18*(b*Cos[c + d
*x])^(5/2)*Sin[c + d*x])/(77*b*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])
/(11*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^4}$$
$$= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^3d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b^2}$$
$$= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77bd} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^3d} + \frac{45}{77}\int (b\cos(c$$
$$= \frac{30b\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77bd} + \frac{2(b\cos(c+dx}{11}$$
$$= \frac{30b\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77bd} + \frac{2(b\cos(c+dx}{11}$$
$$= \frac{30b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30b\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c}{}$$

Mathematica [A] time = 0.199699, size = 83, normalized size = 0.66

$$\frac{(b\cos(c+dx))^{3/2}\left(240F\left(\frac{1}{2}(c+dx)\middle|2\right)+(290\sin(c+dx)+57\sin(3(c+dx))+7\sin(5(c+dx)))\sqrt{\cos(c+dx)}\right)}{616d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(240*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*(290*Sin[c + d*x] + 57*Sin[3*(c + d*x)] + 7*Sin[5*(c + d*x)])))/(616*d*Cos
[c + d*x]^(3/2))
```

Maple [A] time = 1.842, size = 236, normalized size = 1.9

$$-\frac{2b^2}{77d}\sqrt{b\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(448(\cos(1/2dx+c/2))^{13}-1568(\cos(1/2dx+c/2))^{11}+2384(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(448*co
s(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-20
40*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+
15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*
c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*
c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}b\cos(dx+c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

3.79 $\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^2d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}+\frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] (14*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*
x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*(b*Cos[c + d*x]
)^(7/2)*Sin[c + d*x])/(9*b^2*d)
```

Rubi [A] time = 0.0601152, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2640, 2639}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^2d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}+\frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (14*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*
x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*(b*Cos[c + d*x]
)^(7/2)*Sin[c + d*x])/(9*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^3}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d} + \frac{7\int (b\cos(c+dx))^{5/2}\,dx}{9b}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d} + \frac{1}{15}(7b)\int \sqrt{}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d} + \frac{\left(7b\sqrt{b\cos(\right.}{}$$
$$= \frac{14b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos}{}$$

Mathematica [A] time = 0.100421, size = 75, normalized size = 0.79

$$\frac{(b\cos(c+dx))^{3/2}\left(168E\left(\frac{1}{2}(c+dx)\middle|2\right)+(38\sin(2(c+dx))+5\sin(4(c+dx)))\sqrt{\cos(c+dx)}\right)}{180d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(168*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*(38*Sin[2*(c + d*x)] + 5*Sin[4*(c + d*x)])))/(180*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 1.695, size = 223, normalized size = 2.4

$$-\frac{2b^2}{45d}\sqrt{b\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(160(\cos(1/2dx+c/2))^{11}-480(\cos(1/2dx+c/2))^9+616(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(160*co
s(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*c
os(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2
4*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b\cos\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

3.80 $\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=98

$$\frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}+\frac{10b\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}$$

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (10*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.0605094, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}+\frac{10b\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (10*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b^2}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{5}{7}\int (b\cos(c+dx))^{3/2}\,dx$$
$$= \frac{10b\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{1}{21}\left(5b^2\right)\int \frac{}{\sqrt{b}}$$
$$= \frac{10b\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{\left(5b^2\sqrt{\cos(c+d}\right.}{21\sqrt{b\,c}}$$
$$= \frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+}{}$$

Mathematica [A] time = 0.0995934, size = 73, normalized size = 0.74

$$\frac{(b\cos(c+dx))^{3/2}\left(20F\left(\frac{1}{2}(c+dx)\middle|2\right)+(23\sin(c+dx)+3\sin(3(c+dx)))\sqrt{\cos(c+dx)}\right)}{42d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*
(23*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 1.868, size = 210, normalized size = 2.1

$$-\frac{2\,b^2}{21\,d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(48\,(\cos(1/2\,dx+c/2))^9-120\,(\cos(1/2\,dx+c/2))^7+128\,(\cos(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*cos
(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(
1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*
cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b\cos\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.81 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=67

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (6*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.039129, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 2635, 2640, 2639}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (6*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx &= \frac{\int (b\cos(c+dx))^{5/2}\,dx}{b}\\
&= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}(3b)\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{\left(3b\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}\\
&= \frac{6b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] time = 0.098146, size = 65, normalized size = 0.97

$$
\frac{(b\cos(c+dx))^{5/2}\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\sqrt{\cos(c+dx)}\right)}{5bd\cos^{\frac{5}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*S
in[2*(c + d*x)]))/(5*b*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 1.779, size = 213, normalized size = 3.2

$$
-\frac{2b^2}{5d}\sqrt{b\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8(\sin(1/2dx+c/2))^6\cos(1/2dx+c/2)+8(\sin(1/2dx+c/
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-8*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}b\cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

3.82 $\int (b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0333971, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2642, 2641}

$$\frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{3/2}\,dx &= \frac{2b\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{2b\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{\left(b^2\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\ &= \frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0153505, size = 58, normalized size = 0.83

$$\frac{2(b\cos(c+dx))^{3/2}\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*S
in[c + d*x]))/(3*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 2.003, size = 190, normalized size = 2.7

$$-\frac{2b^2}{3d}\sqrt{b\left(2\left(\cos(1/2dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\left(\sin(1/2dx+c/2)\right)^4\cos(1/2dx+c/2)+\sqrt{2\left(\sin(1/2dx+c/2)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(4*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(b\cos(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos(dx+c)}b\cos(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2), x)
```

3.83 $\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx$

Optimal. Leaf size=39

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0291086, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 2640, 2639}

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx &= b\int \sqrt{b\cos(c+dx)}\,dx\\ &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{2b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0151518, size = 39, normalized size = 1.

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 1.563, size = 144, normalized size = 3.7

$$2\,\frac{\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}b^2\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\mathrm{EllipticE}}{\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+c/2))^2\right)}\sin(1/2\,dx+c/2)\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c),x)
```

```
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos(dx+c)}b\cos(dx+c)\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

3.84 $\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]
])
```

Rubi [A] time = 0.0380161, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 2642, 2641}

$$\frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx &= b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{\left(b^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0241456, size = 41, normalized size = 1.

$$\frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]
])
```

Maple [B] time = 1.359, size = 144, normalized size = 3.5

$$-2\frac{\sqrt{b\left(2(\cos(1/2dx+c/2))^2-1\right)(\sin(1/2dx+c/2))^2}b^2\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{-2(\cos(1/2dx+c/2))^2+1}\text{EllipticF}\left(c\right.}{\sqrt{-b\left(2(\sin(1/2dx+c/2))^4-(\sin(1/2dx+c/2))^2\right)}\sin(1/2dx+c/2)\sqrt{b\left(2(\cos(1/2dx+c/2))^2-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/si
n(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}b\cos(dx+c)\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

3.85 $\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

Optimal. Leaf size=66

$$\frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0544, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] (-2*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = b^3\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - b\int \sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.055922, size = 50, normalized size = 0.76

$$\frac{2b^2\left(\sin(c+dx) - \sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(d
*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 1.958, size = 168, normalized size = 2.6

$$-2\,\frac{b^2\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2b}\left(\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\mathrm{EllipticE}\right.}{\sqrt{-b\left(2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4-\left(\sin\left(1/2\,dx+c/2\right)\right)^2\right)}\sin\left(1/2\,dx+c/2\right)\sqrt{b\left(2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x)
```

```
[Out] -2*b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*((sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*
x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}b\cos(dx+c)\sec(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

3.86 $\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0537292, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2642, 2641}

$$\frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx = b^4\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left(b^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$
$$= \frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.067809, size = 51, normalized size = 0.71

$$\frac{2b^2\left(\tan(c+dx)+\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 2.011, size = 241, normalized size = 3.4

$$-\frac{2b^2}{3d}\left(-2\sqrt{2(\sin(1/2dx+c/2))^2-1}\,\mathrm{EllipticF}\left(\cos(1/2dx+c/2),\sqrt{2}\right)\sqrt{(\sin(1/2dx+c/2))^2}(\sin(1/2dx+c/2))^2+\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x)
```

```
[Out] -2/3*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*b^2*(b*(2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/
2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos\left(dx+c\right)}b\cos\left(dx+c\right)\sec\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)*sec(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**4,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)

3.87 $\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx$

Optimal. Leaf size=98

$$\frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-6*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) + (2*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*b^2*Sin[c + d*
x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0737354, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^5,x]
```


```
[Out] (-6*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) + (2*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*b^2*Sin[c + d*
x])/(5*d*Sqrt[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx &= b^5\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(3b^3\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{1}{5}(3b)\int \sqrt{b\cos(c+dx)}\,dx \\
&= \frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3b\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}}{5\sqrt{\cos(c+dx)}} \\
&= -\frac{6b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.125421, size = 69, normalized size = 0.7

$$
\frac{\sec^4(c+dx)(b\cos(c+dx))^{3/2}\left(7\sin(c+dx)+3\sin(3(c+dx))-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^5,x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c
 + d*x)/2, 2] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Maple [B] time = 3.349, size = 364, normalized size = 3.7

$$
\frac{2b}{5d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d*x
+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1
/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2
+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}b\cos(dx+c)\sec(dx+c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)*sec(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

3.88 $\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^3\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (2*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b^3*Sin[c
+ d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0734373, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2642, 2641}

$$\frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^3\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^6,x]
```

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (2*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b^3*Sin[c
+ d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx = b^6\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(5b^4\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^3\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{1}{21}\left(5b^2\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^3\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{\left(5b^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}$$
$$= \frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^3\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.12899, size = 69, normalized size = 0.69

$$\frac{\sec^4(c+dx)(b\cos(c+dx))^{3/2}\left(5\sin(2(c+dx))+6\tan(c+dx)+10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^6,x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*(10*Cos[c + d*x]^(5/2)*EllipticF[(c
+ d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Maple [B] time = 1.856, size = 398, normalized size = 4.

$$-\frac{2\,b^2}{21\,d}\left(-40\,\sqrt{(\sin(1/2\,dx+c/2))^2}\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,(\sin(1/2\,dx+c/2))^6+6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-3
0*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c))*b^2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b\cos\left(dx+c\right)\sec\left(dx+c\right)^{6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)*sec(d*x + c)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

3.89 $\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx$

Optimal. Leaf size=126

$$\frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b^2\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] (-14*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d
*x]]) + (2*b^6*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^4*Sin[c +
 d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*b^2*Sin[c + d*x])/(15*d*Sqrt[b*C
os[c + d*x]])
```

Rubi [A] time = 0.0945455, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b^2\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^7,x]
```


```
[Out] (-14*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d
*x]]) + (2*b^6*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^4*Sin[c +
 d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*b^2*Sin[c + d*x])/(15*d*Sqrt[b*C
os[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx = b^7\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}\left(7b^5\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{1}{15}\left(7b^3\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b^2\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} - \frac{1}{15}(7b)\int$$
$$= \frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b^2\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} - \left(7b\sqrt{b\cos}\right.$$
$$= -\frac{14b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^4\sin(c+d}{45d(b\cos(c+dx}$$

Mathematica [A] time = 0.19001, size = 79, normalized size = 0.63

$$\frac{\sec^6(c+dx)(b\cos(c+dx))^{3/2}\left(150\sin(c+dx)+91\sin(3(c+dx))+21\sin(5(c+dx))-336\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\right.\right.}{360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^7,x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^6*(-336*Cos[c + d*x]^(9/2)*EllipticE[(
c + d*x)/2, 2] + 150*Sin[c + d*x] + 91*Sin[3*(c + d*x)] + 21*Sin[5*(c + d*x
)]))/(360*d)
```

Maple [B] time = 3.475, size = 414, normalized size = 3.3

$$-2\frac{\sqrt{b\left(2(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}b^2}{\sin(1/2\,dx+c/2)\sqrt{b\left(2(\cos(1/2\,dx+c/2))^2-1\right)}d}\left(-\frac{\cos(1/2\,dx+c/2)\sqrt{-b\left(2(\sin(1/2\,dx+c/2))^4-(\sin(1/2\right.}}{144\,b\left((\cos(1/2\,dx+c/2))^2-1/2\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-1/144*co
s(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/
2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}} \sec(dx+c)^7 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}b\cos(dx+c)\sec(dx+c)^7, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b*cos(d*x + c)*sec(d*x + c)^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{3}{2}} \sec(dx+c)^7 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^7, x)
```

3.90 $\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=125

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^2d}+\frac{30b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{77d}+\frac{30b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}}+\frac{18\sin(c+d}{}$$

```
[Out] (30*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c +
d*x]]) + (30*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*d) + (18*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(77*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])
/(11*b^2*d)
```

Rubi [A] time = 0.0804959, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^2d}+\frac{30b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{77d}+\frac{30b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}}+\frac{18\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (30*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c +
d*x]]) + (30*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*d) + (18*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(77*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])
/(11*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^3}$$
$$= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^2d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b}$$
$$= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^2d} + \frac{1}{77}(45b)\int (b\,c$$
$$= \frac{30b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77d} + \frac{2(b\cos(c+d}{1}$$
$$= \frac{30b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77d} + \frac{2(b\cos(c+d}{1}$$
$$= \frac{30b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c}{}$$

Mathematica [A] time = 0.195242, size = 83, normalized size = 0.66

$$\frac{(b\cos(c+dx))^{5/2}\left(240F\left(\frac{1}{2}(c+dx)\middle|2\right)+(290\sin(c+dx)+57\sin(3(c+dx))+7\sin(5(c+dx)))\sqrt{\cos(c+dx)}\right)}{616d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(240*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*(290*Sin[c + d*x] + 57*Sin[3*(c + d*x)] + 7*Sin[5*(c + d*x)])))/(616*d*Cos
[c + d*x]^(5/2))
```

Maple [A] time = 2.044, size = 236, normalized size = 1.9

$$-\frac{2b^3}{77d}\sqrt{b\left(2(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(448(\cos(1/2\,dx+c/2))^{13}-1568(\cos(1/2\,dx+c/2))^{11}+2384\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(448*co
s(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-20
40*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+
15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*
c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*
c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b^2\cos\left(dx+c\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

3.91 $\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}+\frac{14b\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}$$

```
[Out] (14*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c +
d*x]]) + (14*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*(b*Cos[c +
d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.0589, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2640, 2639}

$$\frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}+\frac{14b\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (14*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c +
d*x]]) + (14*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*(b*Cos[c +
d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^2}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{7}{9}\int (b\cos(c+dx))^{5/2}\,dx$$
$$= \frac{14b(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{1}{15}\left(7b^2\right)\int$$
$$= \frac{14b(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{\left(7b^2\sqrt{b\,\mathrm{co}\right.}{}$$
$$= \frac{14b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{14b(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\,\mathrm{c}}{}$$

Mathematica [A] time = 0.101057, size = 75, normalized size = 0.77

$$\frac{(b\cos(c+dx))^{5/2}\left(168E\left(\frac{1}{2}(c+dx)\middle|2\right)+(38\sin(2(c+dx))+5\sin(4(c+dx)))\sqrt{\cos(c+dx)}\right)}{180d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(168*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*(38*Sin[2*(c + d*x)] + 5*Sin[4*(c + d*x)])))/(180*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 1.743, size = 223, normalized size = 2.3

$$-\frac{2\,b^3}{45\,d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(160\,(\cos(1/2\,dx+c/2))^{11}-480\,(\cos(1/2\,dx+c/2))^9+616\,(\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(160*co
s(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*c
os(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2
4*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b^2\cos\left(dx+c\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

3.92 $\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=97

$$\frac{10b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

```
[Out] (10*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (10*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.0579795, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 2635, 2642, 2641}

$$\frac{10b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (10*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (10*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}(5b)\int (b\cos(c+dx))^{3/2}\,dx$$
$$= \frac{10b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{21}\left(5b^3\right)\int \frac{}{\sqrt{b}}$$
$$= \frac{10b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{\left(5b^3\sqrt{\cos(c+d}\right.}{21\sqrt{b\,c}}$$
$$= \frac{10b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+}{}$$

Mathematica [A] time = 0.0947634, size = 76, normalized size = 0.78

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(20F\left(\frac{1}{2}(c+dx)\middle|2\right)+(23\sin(c+dx)+3\sin(3(c+dx)))\sqrt{\cos(c+dx)}\right)}{42d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(23*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.015, size = 210, normalized size = 2.2

$$-\frac{2b^3}{21d}\sqrt{b\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(48(\cos(1/2dx+c/2))^9-120(\cos(1/2dx+c/2))^7+128(\cos(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(48*cos
(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(
1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*
cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b^2\cos\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.93 $\int (b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

```
[Out] (6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0327125, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2640, 2639}

$$\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (b\cos(c+dx))^{5/2}\,dx &= \frac{2b(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}\left(3b^2\right)\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2b(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{\left(3b^2\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}\\
&= \frac{6b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.058777, size = 62, normalized size = 0.89

$$\frac{(b\cos(c+dx))^{5/2}\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\sqrt{\cos(c+dx)}\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*S
in[2*(c + d*x)]))/(5*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 1.883, size = 213, normalized size = 3.

$$-\frac{2b^3}{5d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+8\left(\sin(1/2\,dx+c/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-8*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}b^2\cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2), x)
```

3.94 $\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{2b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0481798, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 2635, 2642, 2641}

$$\frac{2b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] (2*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx &= b\int (b\cos(c+dx))^{3/2}\,dx\\
&= \frac{2b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}b^3\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{2b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{\left(b^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\
&= \frac{2b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] time = 0.0242718, size = 59, normalized size = 0.82

$$\frac{2b(b\cos(c+dx))^{3/2}\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] (2*b*(b*Cos[c + d*x])^(3/2)*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*Sin[c + d*x]))/(3*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 1.786, size = 190, normalized size = 2.6

$$-\frac{2b^3}{3d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+\sqrt{2\left(\sin(1/2\,dx+c/2)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c),x)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(4*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b^2\cos\left(dx+c\right)^2\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^2*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

3.95 $\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
])
```

Rubi [A] time = 0.0394378, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 2640, 2639}

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx &= b^2\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{2b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0145408, size = 41, normalized size = 1.

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
])
```

Maple [B] time = 1.441, size = 144, normalized size = 3.5

$$2\frac{\sqrt{b\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}b^{3}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\mathrm{EllipticE}}{\sqrt{-b\left(2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}-\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}\right)}\sin\left(1/2\,dx+c/2\right)\sqrt{b\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x)
```

```
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos\left(dx+c\right)}b^{2}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^2*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

3.96 $\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]
])
```

Rubi [A] time = 0.0380636, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 2642, 2641}

$$\frac{2b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```


```
[Out] (2*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]
])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx &= b^3\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{\left(b^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{2b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0437825, size = 38, normalized size = 0.93

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(b\cos(c+dx))^{5/2}}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(5/2)*EllipticF[(c + d*x)/2, 2])/(d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 1.421, size = 144, normalized size = 3.5

$$-2\frac{\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^{2}-1\right)\left(\sin(1/2\,dx+c/2)\right)^{2}}b^{3}\sqrt{\left(\sin(1/2\,dx+c/2)\right)^{2}}\sqrt{-2\left(\cos(1/2\,dx+c/2)\right)^{2}+1}\mathrm{EllipticF}\left(c\right.}{\sqrt{-b\left(2\left(\sin(1/2\,dx+c/2)\right)^{4}-\left(\sin(1/2\,dx+c/2)\right)^{2}\right)}\sin(1/2\,dx+c/2)\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^{2}-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/si
n(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos(dx+c)}b^{2}\cos(dx+c)^{2}\sec(dx+c)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^2*sec(d*x + c)^3, x)
```

<u>Sympy</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

3.97 $\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

Optimal. Leaf size=68

$$\frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.055213, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] (-2*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = b^4\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - b^2\int \sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0583077, size = 50, normalized size = 0.74

$$\frac{2b^3\left(\sin(c+dx)-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^3*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(d
*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.019, size = 168, normalized size = 2.5

$$-2\,\frac{b^3\sqrt{-2\,b\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2\,b}\left(\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\right.}{\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+c/2))^2\right)}\sin(1/2\,dx+c/2)\sqrt{b(2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x)
```

```
[Out] -2*b^3*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*((sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*
x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}b^2\cos(dx+c)^2\sec(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^2*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

3.98 $\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0563395, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2642, 2641}

$$\frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

```
[Out] (2*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx = b^5\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}b^3\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left(b^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$
$$= \frac{2b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0698673, size = 51, normalized size = 0.71

$$\frac{2b^3\left(\tan(c+dx)+\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

```
[Out] (2*b^3*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 1.871, size = 241, normalized size = 3.4

$$-\frac{2b^3}{3d}\left(-2\sqrt{2(\sin(1/2dx+c/2))^2-1}\text{EllipticF}\left(\cos(1/2dx+c/2),\sqrt{2}\right)\sqrt{(\sin(1/2dx+c/2))^2}(\sin(1/2dx+c/2))^2+\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x)
```

```
[Out] -2/3*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*b^3*(b*(2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/
2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b^{2}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^2*sec(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

3.99 $\int (b\cos (c+dx))^{5/2}\sec ^6(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{2b^5\sin (c+dx)}{5d(b\cos (c+dx))^{5/2}}+\frac{6b^3\sin (c+dx)}{5d\sqrt{b\cos (c+dx)}}-\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos (c+dx)}}{5d\sqrt{\cos (c+dx)}}$$

```
[Out] (-6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*b^3*Sin[c +
d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.07335, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^5\sin (c+dx)}{5d(b\cos (c+dx))^{5/2}}+\frac{6b^3\sin (c+dx)}{5d\sqrt{b\cos (c+dx)}}-\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos (c+dx)}}{5d\sqrt{\cos (c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^6,x]
```


```
[Out] (-6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*b^3*Sin[c +
d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx = b^6\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(3b^4\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{1}{5}\left(3b^2\right)\int \sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3b^2\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx}}{5\sqrt{\cos(c+dx)}}$$
$$= -\frac{6b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx}}$$

Mathematica [A] time = 0.156837, size = 69, normalized size = 0.69

$$\frac{\sec^5(c+dx)(b\cos(c+dx))^{5/2}\left(7\sin(c+dx)+3\sin(3(c+dx))-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^6,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c
 + d*x)/2, 2] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Maple [B] time = 3.274, size = 366, normalized size = 3.7

$$\frac{2b^2}{5d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2*d
*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b^{2}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^2*sec(d*x + c)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

3.100 $\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^4\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[Out] (10*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (2*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b^4*Sin[c
+ d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0741106, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2642, 2641}

$$\frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^4\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^7,x]
```

```
[Out] (10*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (2*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b^4*Sin[c
+ d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx = b^7\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx$$
$$= \frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(5b^5\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^4\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{1}{21}\left(5b^3\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^4\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{\left(5b^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}$$
$$= \frac{10b^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^4\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.177607, size = 69, normalized size = 0.69

$$\frac{\sec^5(c+dx)(b\cos(c+dx))^{5/2}\left(5\sin(2(c+dx))+6\tan(c+dx)+10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^7,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(10*Cos[c + d*x]^(5/2)*EllipticF[(c
+ d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Maple [B] time = 1.819, size = 398, normalized size = 4.

$$-\frac{2b^3}{21d}\left(-40\sqrt{(\sin(1/2\,dx+c/2))^2}\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,(\sin(1/2\,dx+c/2))^6+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-3
0*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c))*b^3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b^2\cos\left(dx+c\right)^2\sec\left(dx+c\right)^7, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^2*sec(d*x + c)^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

3.101 $\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx$

Optimal. Leaf size=128

$$\frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b^3\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] (-14*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c +
 d*x]]) + (2*b^7*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^5*Sin[c
 + d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*b^3*Sin[c + d*x])/(15*d*Sqrt[b
*Cos[c + d*x]])
```

Rubi [A] time = 0.0948979, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b^3\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^8,x]
```

```
[Out] (-14*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c +
 d*x]]) + (2*b^7*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^5*Sin[c
 + d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*b^3*Sin[c + d*x])/(15*d*Sqrt[b
*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx = b^8\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}\left(7b^6\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{1}{15}\left(7b^4\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b^3\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} - \frac{1}{15}\left(7b^2\right)\int$$
$$= \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b^3\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} - \left(7b^2\sqrt{b\cos}\right.$$
$$= -\frac{14b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^5\sin(c+}{45d(b\cos(c+d}$$

Mathematica [A] time = 0.25978, size = 79, normalized size = 0.62

$$\frac{\sec^7(c+dx)(b\cos(c+dx))^{5/2}\left(150\sin(c+dx)+91\sin(3(c+dx))+21\sin(5(c+dx))-336\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\right.\right.}{360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^8,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^7*(-336*Cos[c + d*x]^(9/2)*EllipticE[(
c + d*x)/2, 2] + 150*Sin[c + d*x] + 91*Sin[3*(c + d*x)] + 21*Sin[5*(c + d*x
)]))/(360*d)
```

Maple [B] time = 3.356, size = 414, normalized size = 3.2

$$-2\,\frac{\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}b^3}{\sin(1/2\,dx+c/2)\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)}d}\left(-\frac{\cos(1/2\,dx+c/2)\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\right.}}{144\,b\left((\cos(1/2\,dx+c/2))^2-1/2\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-1/144*co
s(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/
2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)}b^2\cos(dx+c)^2\sec(dx+c)^8\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)^2*sec(d*x + c)^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^8, x)
```

3.102 $\int (b\cos(c+dx))^{7/2}\,dx$

Optimal. Leaf size=98

$$\frac{10b^3\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10b^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

```
[Out] (10*b^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (10*b^3*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(b*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.0505346, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2642, 2641}

$$\frac{10b^3\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10b^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (10*b^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (10*b^3*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(b*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{7/2}\,dx = \frac{2b(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}\left(5b^2\right)\int (b\cos(c+dx))^{3/2}\,dx$$
$$= \frac{10b^3\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{21}\left(5b^4\right)\int \frac{1}{\sqrt{b\cos(c+dx}}$$
$$= \frac{10b^3\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{\left(5b^4\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(}}}{21\sqrt{b\cos(c+dx)}}$$
$$= \frac{10b^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b^3\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b(b\cos(c+dx))^{5/2}\sin(}{7d}$$

Mathematica [A] time = 0.0209971, size = 76, normalized size = 0.78

$$\frac{b^3\sqrt{b\cos(c+dx)}\left(20F\left(\frac{1}{2}(c+dx)\middle|2\right)+(23\sin(c+dx)+3\sin(3(c+dx)))\sqrt{\cos(c+dx)}\right)}{42d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (b^3*Sqrt[b*Cos[c + d*x]]*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(23*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 1.878, size = 210, normalized size = 2.1

$$-\frac{2\,b^4}{21\,d}\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(48\,\left(\cos\left(1/2\,dx+c/2\right)\right)^9-120\,\left(\cos\left(1/2\,dx+c/2\right)\right)^7+128\,\left(\cos\left(1/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*(48*cos
(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(
1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*
cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)}b^3\cos\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*b^3*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.103 $\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=125

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^5d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^3d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77bd}+\frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{77d\sqrt{b\cos(c+}$$

```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c + d*x]
]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*b*d) + (18*(b*Cos[c + d*x])
^(5/2)*Sin[c + d*x])/(77*b^3*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(
11*b^5*d)
```

Rubi [A] time = 0.0913516, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^5d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^3d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77bd}+\frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{77d\sqrt{b\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/Sqrt[b*Cos[c + d*x]],x]
```


```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c + d*x]
]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*b*d) + (18*(b*Cos[c + d*x])
^(5/2)*Sin[c + d*x])/(77*b^3*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(
11*b^5*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^6}$$
$$= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b^4}$$
$$= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d} + \frac{45\int (b\cos(c+dx))^{3/2}\,dx}{77b^2}$$
$$= \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77bd} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+}{11b^5d}$$
$$= \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77bd} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+}{11b^5d}$$
$$= \frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77bd} + \frac{18(b\cos(c+dx))^{5/2}\sin(c}{77b^3d}$$

Mathematica [A] time = 0.141639, size = 73, normalized size = 0.58

$$\frac{347\sin(2(c+dx))+64\sin(4(c+dx))+7\sin(6(c+dx))+480\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{1232d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (480*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 347*Sin[2*(c + d*x)] +
64*Sin[4*(c + d*x)] + 7*Sin[6*(c + d*x)])/(1232*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.155, size = 233, normalized size = 1.9

$$-\frac{2}{77\,d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(448\,(\cos(1/2\,dx+c/2))^{13}-1568\,(\cos(1/2\,dx+c/2))^{11}+238\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(448*cos(1/
2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-2040*c
os(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+15*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2
-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{5}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^5/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{6}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/sqrt(b*cos(d*x + c)), x)
```

3.104 $\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^4d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^2d}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*
x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^2*d) + (2*(b*Cos[c +
d*x])^(7/2)*Sin[c + d*x])/(9*b^4*d)
```

Rubi [A] time = 0.0589798, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2640, 2639}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^4d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^2d}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*
x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^2*d) + (2*(b*Cos[c +
d*x])^(7/2)*Sin[c + d*x])/(9*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^5}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{7\int (b\cos(c+dx))^{5/2}\,dx}{9b^3}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{7\int \sqrt{b\cos(c+dx)}\,dx}{15b}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{\left(7\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c}{15b\sqrt{\cos(c+dx)}}$$
$$= \frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+}{9b^4d}$$

Mathematica [A] time = 0.122968, size = 71, normalized size = 0.71

$$\frac{(38\sin(2(c+dx))+5\sin(4(c+dx)))\cos(c+dx)+168\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{180d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (168*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*(38*Sin[2*
(c + d*x)] + 5*Sin[4*(c + d*x)]))/(180*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 1.894, size = 220, normalized size = 2.2

$$-\frac{2}{45d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(160\left(\cos(1/2\,dx+c/2)\right)^{11}-480\left(\cos(1/2\,dx+c/2)\right)^9+616\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/
2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*cos(1
/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*co
s(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^5}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\cos(dx+c)^4}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^4/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^5}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

3.105 $\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=97

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21bd}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]
]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d) + (2*(b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(7*b^3*d)
```

Rubi [A] time = 0.0617787, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21bd}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]
]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d) + (2*(b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(7*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b^4}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{5\int (b\cos(c+dx))^{3/2}\,dx}{7b^2}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{5}{21}\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{\left(5\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx}}}{21\sqrt{b\cos(c+dx)}}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+}{7b^3d}$$

Mathematica [A] time = 0.10518, size = 63, normalized size = 0.65

$$\frac{26\sin(2(c+dx)) + 3\sin(4(c+dx)) + 40\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{84d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26*Sin[2*(c + d*x)] + 3*
Sin[4*(c + d*x)])/(84*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 1.7, size = 207, normalized size = 2.1

$$-\frac{2}{21\,d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(48\,(\cos(1/2\,dx+c/2))^9 - 120\,(\cos(1/2\,dx+c/2))^7 + 128\,(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*cos(1/2
*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(1/2*
d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*sin(
1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(
1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\cos(dx+c)^3}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^3/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

3.106 $\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]
]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)
```

Rubi [A] time = 0.039397, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2640, 2639}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]
]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{5/2}\,dx}{b^3}$$
$$= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b}$$
$$= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}}$$
$$= \frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}$$

Mathematica [A] time = 0.0564639, size = 58, normalized size = 0.81

$$\frac{\sin(2(c+dx))\cos(c+dx)+6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*Sin[2*(c + d
*x)])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 1.682, size = 210, normalized size = 2.9

$$-\frac{2}{5\,d}\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8\,\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+8\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2
*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2
*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\cos(dx+c)^2}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^2/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.107 $\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=69

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.039005, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\,dx}{b^2}$$
$$= \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{1}{3}\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}$$
$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

Mathematica [A] time = 0.0450701, size = 51, normalized size = 0.74

$$\frac{\sin(2(c+dx)) + 2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])/(3*d*Sq
rt[b*Cos[c + d*x]])
```

Maple [B] time = 1.863, size = 187, normalized size = 2.7

$$-\frac{2}{3d}\sqrt{b\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\left(\sin\left(1/2\,dx+c/2\right)\right)^4\cos\left(1/2\,dx+c/2\right)+\sqrt{2\left(\sin\left(1/2\,dx+c/\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\cos(dx+c)}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.108 $\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=41

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0234976, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 2640, 2639}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\int \sqrt{b\cos(c+dx)}\,dx}{b}\\ &= \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}}\\ &= \frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0150819, size = 41, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 1.409, size = 141, normalized size = 3.4

$$2\,{\frac{\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}{\rm EllipticE}\left(\cos\left(\right.\right.}{\sqrt{-b\left(2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}-\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}\right)}\sin\left(1/2\,dx+c/2\right)\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2
*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.109 $\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0192094, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2642, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0151406, size = 38, normalized size = 1.

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [C] time = 0.2, size = 54, normalized size = 1.4

$$2\,{\frac{\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}{\rm InverseJacobiAM}\left(1/2\,dx+c/2,\sqrt{2}\right)}{d\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*I
nverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \cos\left(dx + c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{b \cos\left(dx + c\right)}}{b \cos\left(dx + c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \cos\left(c + dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c)), x)
```

3.110 $\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=65

$$\frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0444571, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 2636, 2640, 2639}

$$\frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}dx &= b\int \frac{1}{(b\cos(c+dx))^{3/2}}dx\\ &= \frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b}\\ &= \frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b\sqrt{\cos(c+dx)}}\\ &= -\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0434025, size = 47, normalized size = 0.72

$$\frac{2\left(\sin(c+dx)-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(d*Sqr
t[b*Cos[c + d*x]])
```

Maple [A] time = 2.019, size = 165, normalized size = 2.5

$$-2\frac{\sqrt{-2b(\sin(1/2dx+c/2))^4+(\sin(1/2dx+c/2))^2b}\left(\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{2(\sin(1/2dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos\right.\right.}{\sqrt{-b\left(2(\sin(1/2dx+c/2))^4-(\sin(1/2dx+c/2))^2\right)}\sin(1/2dx+c/2)\sqrt{b\left(2(\cos\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/
2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/
2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{b\cos(dx+c)}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)}{b\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(c+dx\right)}{\sqrt{b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.111 $\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0514868, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2642, 2641}

$$\frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\ &= \frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0640158, size = 48, normalized size = 0.72

$$\frac{2\left(\tan(c+dx)+\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d*Sqrt
[b*Cos[c + d*x]])
```

Maple [B] time = 1.994, size = 238, normalized size = 3.6

$$-\frac{2}{3d}\left(-2\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(\sin\left(1/2\,dx+c/2\right)\right)^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(b*(2*cos(1/2*d*x+1/2*c)^2-
1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*
x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{2}}{b\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{2}\left(c+dx\right)}{\sqrt{b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.112 $\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=97

$$\frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x
]]) + (2*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/
(5*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0732165, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x
]]) + (2*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/
(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b^3\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}(3b)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{3\int\sqrt{b\cos(c+dx)}\,dx}{5b}$$
$$= \frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}}$$
$$= -\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.098034, size = 65, normalized size = 0.67

$$\frac{6\sin(c+dx) - 6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\tan(c+dx)\sec(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[c
 + d*x]*Tan[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 3.485, size = 366, normalized size = 3.8

$$\frac{2}{5\,bd}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x
+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1
/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2
+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maximा [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{3}}{b\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^3/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}\left(c+dx\right)}{\sqrt{b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{3}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.113 $\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]
]) + (2*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b*Sin[c + d*x]
)/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0718351, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2642, 2641}

$$\frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]
]) + (2*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b*Sin[c + d*x]
)/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b^4\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(5b^2\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{5}{21}\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{\left(5\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\sqrt{b\cos(c+dx)}}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.143704, size = 63, normalized size = 0.66

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\tan(c+dx)\left(3\sec^2(c+dx)+5\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5 + 3*Sec[c + d*x]^2)
*Tan[c + d*x])/(21*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 1.967, size = 395, normalized size = 4.2

$$-\frac{2}{21\,d}\left(-40\,\sqrt{(\sin(1/2\,dx+c/2))^2}\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,(\sin(1/2\,dx+c/2))^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-3
0*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c))*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(
2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{4}}{b\cos\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^4/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{4}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

3.114 $\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=125

$$\frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$

```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d
*x]]) + (2*b^4*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^2*Sin[c +
 d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*Sin[c + d*x])/(15*d*Sqrt[b*Cos[c
 + d*x]])
```

Rubi [A] time = 0.0937142, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```


```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d
*x]]) + (2*b^4*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^2*Sin[c +
 d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*Sin[c + d*x])/(15*d*Sqrt[b*Cos[c
 + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b^5\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}\left(7b^3\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{1}{15}(7b)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} - \frac{7\int\sqrt{b\cos(c+dx)}\,dx}{15b}$$
$$= \frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} - \frac{\left(7\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c}}{15b\sqrt{\cos(c+dx)}}$$
$$= -\frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c}{15d\sqrt{b\cos(}}$$

Mathematica [A] time = 0.315153, size = 77, normalized size = 0.62

$$\frac{42\sin(c+dx) - 42\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\tan(c+dx)\sec(c+dx)\left(5\sec^2(c+dx)+7\right)}{45d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-42*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 2*Sec
[c + d*x]*(7 + 5*Sec[c + d*x]^2)*Tan[c + d*x])/(45*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 3.677, size = 411, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/72*cos(1/2*d
*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^5-7/90*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^
4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-28/15*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/
2*c)^2)^(1/2)+14/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-14/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\sec(dx+c)^5}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^5/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

3.115 $\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^6d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^4d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77b^2d}+\frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{77bd\sqrt{b\cos(c}}$$

```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*b*d*Sqrt[b*Cos[c + d*
x]]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*b^2*d) + (18*(b*Cos[c + d
*x])^(5/2)*Sin[c + d*x])/(77*b^4*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x
])/(11*b^6*d)
```

Rubi [A] time = 0.0825005, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^6d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^4d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77b^2d}+\frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{77bd\sqrt{b\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*b*d*Sqrt[b*Cos[c + d*
x]]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*b^2*d) + (18*(b*Cos[c + d
*x])^(5/2)*Sin[c + d*x])/(77*b^4*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x
])/(11*b^6*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^7}$$
$$= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^6d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b^5}$$
$$= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^4d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^6d} + \frac{45\int (b\cos(c+dx))^{3/2}\,d}{77b^3}$$
$$= \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77b^2d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^4d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c}{11b^6d}$$
$$= \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77b^2d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^4d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c}{11b^6d}$$
$$= \frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77bd\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77b^2d} + \frac{18(b\cos(c+dx))^{5/2}\sin(}{77b^4d}$$

Mathematica [A] time = 0.115486, size = 76, normalized size = 0.59

$$\frac{347\sin(2(c+dx)) + 64\sin(4(c+dx)) + 7\sin(6(c+dx)) + 480\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{1232bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (480*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 347*Sin[2*(c + d*x)] +
64*Sin[4*(c + d*x)] + 7*Sin[6*(c + d*x)])/(1232*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.177, size = 236, normalized size = 1.8

$$-\frac{2}{77\,bd}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(448\,(\cos(1/2\,dx+c/2))^{13}-1568\,(\cos(1/2\,dx+c/2))^{11}+23\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(448*cos(
1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-2040
*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+15
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)
^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^7}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^5}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^5/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^7}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(3/2), x)
```

3.116 $\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^5d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^3d}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c +
d*x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^3*d) + (2*(b*Cos[c
+ d*x])^(7/2)*Sin[c + d*x])/(9*b^5*d)
```

Rubi [A] time = 0.0580826, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2640, 2639}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^5d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^3d}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c +
d*x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^3*d) + (2*(b*Cos[c
+ d*x])^(7/2)*Sin[c + d*x])/(9*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^6}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{7\int (b\cos(c+dx))^{5/2}\,dx}{9b^4}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{7\int \sqrt{b\cos(c+dx)}\,dx}{15b^2}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{\left(7\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos}}{15b^2\sqrt{\cos(c+dx}}$$
$$= \frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2(b\cos(c+dx))^{7/2}\sin(}{9b^5d}$$

Mathematica [A] time = 0.112998, size = 74, normalized size = 0.74

$$\frac{(38\sin(2(c+dx)) + 5\sin(4(c+dx)))\cos(c+dx) + 168\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{180bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (168*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*(38*Sin[2*
(c + d*x)] + 5*Sin[4*(c + d*x)]))/(180*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 1.93, size = 223, normalized size = 2.2

$$-\frac{2}{45\,bd}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(160\,(\cos(1/2\,dx+c/2))^{11} - 480\,(\cos(1/2\,dx+c/2))^9 + 616\,(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(160*cos(
1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*cos
(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*
cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{4}}{b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^4/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{6}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(3/2), x)
```

3.117 $\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*
x]]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*(b*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*b^4*d)
```

Rubi [A] time = 0.0602144, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*
x]]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*(b*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b^5}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{5\int (b\cos(c+dx))^{3/2}\,dx}{7b^3}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{5\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{\left(5\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}}}{21b\sqrt{b\cos(c+dx)}}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c}{7b^4d}$$

Mathematica [A] time = 0.0818611, size = 66, normalized size = 0.66

$$\frac{26\sin(2(c+dx)) + 3\sin(4(c+dx)) + 40\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{84bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26*Sin[2*(c + d*x)] + 3*
Sin[4*(c + d*x)])/(84*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 1.99, size = 210, normalized size = 2.1

$$-\frac{2}{21\,bd}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(48\,(\cos(1/2\,dx+c/2))^9-120\,(\cos(1/2\,dx+c/2))^7+128\,(\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(48*cos(1
/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(1/
2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*co
s(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{3}}{b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^3/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{5}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(3/2), x)
```

3.118 $\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*
x]]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)
```

Rubi [A] time = 0.0396234, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2640, 2639}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*
x]]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int (b\cos(c+dx))^{5/2}\,dx}{b^4} \\
&= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b^2} \\
&= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}} \\
&= \frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}
\end{aligned}
$$

Mathematica [A] time = 0.0524571, size = 61, normalized size = 0.85

$$
\frac{\sin(2(c+dx))\cos(c+dx)+6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*Sin[2*(c + d
*x)])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 2.039, size = 213, normalized size = 3.

$$
-\frac{2}{5\,bd}\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8\,\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+8\,\left(\sin\left(1/2\,dx+c/2\right)\right)\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-8*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*
(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*
(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\cos\left(dx+c\right)^4}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{2}}{b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^2/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{4}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

3.119 $\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.0406043, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\,dx}{b^3}$$
$$= \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b}$$
$$= \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}}$$
$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}$$

Mathematica [A] time = 0.0423844, size = 54, normalized size = 0.75

$$\frac{\sin(2(c+dx)) + 2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])/(3*b*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 1.93, size = 190, normalized size = 2.6

$$-\frac{2}{3\,bd}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,(\sin(1/2\,dx+c/2))^4\cos(1/2\,dx+c/2)+\sqrt{2\,(\sin(1/2\,dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(4*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

3.120 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]
])
```

Rubi [A] time = 0.0233109, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 2640, 2639}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int \sqrt{b\cos(c+dx)}\,dx}{b^2}\\ &= \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}}\\ &= \frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0233004, size = 41, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]
])
```

Maple [B] time = 1.325, size = 144, normalized size = 3.5

$$2\,\frac{\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\mathrm{EllipticE}\left(\cos\left(\right.\right.}{b\sqrt{-b\left(2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}-\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}\right)}\sin\left(1/2\,dx+c/2\right)\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/b/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}}{b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.121 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0238076, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 2642, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b}\\
&= \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0197367, size = 41, normalized size = 1.

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 1.46, size = 144, normalized size = 3.5

$$-2\frac{\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/2\,dx+c/2)\right)^2+1}\text{EllipticF}\,(}{b\sqrt{-b\left(2\left(\sin(1/2\,dx+c/2)\right)^4-\left(\sin(1/2\,dx+c/2)\right)^2\right)}\sin(1/2\,dx+c/2)\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c
)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/sin(
1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}}{b^2\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.122 $\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x
]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0336752, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2640, 2639}

$$\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x
]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}\,dx}{b^2}\\
&= \frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}}\\
&= -\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0283512, size = 50, normalized size = 0.74

$$\frac{2\left(\sin(c+dx)-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(b*d*S
qrt[b*Cos[c + d*x]])
```

Maple [A] time = 1.992, size = 168, normalized size = 2.5

$$-2\,\frac{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b}\left(\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\mathrm{EllipticE}\left(\cos\left(\right.\right.\right.}{b\sqrt{-b\left(2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}-\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}\right)}\sin\left(1/2\,dx+c/2\right)\sqrt{b\left(2\,(\mathrm{c}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*((sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+
1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+
1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}}{b^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(-3/2), x)
```

3.123 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0477973, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 2636, 2642, 2641}

$$\frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b} \\
&= \frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}} \\
&= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0263353, size = 51, normalized size = 0.74

$$
\frac{2\left(\tan(c+dx)+\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3bd\sqrt{b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*b*d*Sq
rt[b*Cos[c + d*x]])
```

Maple [B] time = 1.894, size = 241, normalized size = 3.5

$$
-\frac{2}{3\,bd}\left(-2\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}+\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/b*(b*(2*cos(1/2*d*x+1/2*c)^
2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*
d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)}{b^2\cos\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.124 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d
*x]]) + (2*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/
(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0761207, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d
*x]]) + (2*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/
(5*b*d*Sqrt[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = b^2\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{3}{5}\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} - \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b^2}$$
$$= \frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} - \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}}$$
$$= -\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0806334, size = 68, normalized size = 0.69

$$\frac{6\sin(c+dx) - 6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\tan(c+dx)\sec(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[c
 + d*x]*Tan[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 3.25, size = 366, normalized size = 3.7

$$\frac{2}{5\,b^2d}\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2*d
*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{2}}{b^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.125 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*Sin[c + d*x]
)/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0747481, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2642, 2641}

$$\frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*Sin[c + d*x]
)/(21*d*(b*Cos[c + d*x])^(3/2))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx \\
&= \frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}(5b)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{5\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b} \\
&= \frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{\left(5\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b\sqrt{b\cos(c+dx)}} \\
&= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0885749, size = 66, normalized size = 0.68

$$
\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\tan(c+dx)\left(3\sec^2(c+dx)+5\right)}{21bd\sqrt{b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5 + 3*Sec[c + d*x]^2)
*Tan[c + d*x])/(21*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 2.064, size = 398, normalized size = 4.1

$$
-\frac{2}{21\,bd}\left(-40\,\sqrt{(\sin(1/2\,dx+c/2))^2}\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,(\sin(1/2\,dx+c/2))^6
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-3
0*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c))/b*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)^3}{(b \cos (dx + c))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \cos (dx + c)} \sec (dx + c)^3}{b^2 \cos (dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^3/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)^3}{(b \cos (dx + c))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

3.126 $\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}}+\frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c +
 d*x]]) + (2*b^3*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b*Sin[c +
 d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*Sin[c + d*x])/(15*b*d*Sqrt[b*Cos
[c + d*x]])
```

Rubi [A] time = 0.0961565, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}}+\frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c +
 d*x]]) + (2*b^3*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b*Sin[c +
 d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*Sin[c + d*x])/(15*b*d*Sqrt[b*Cos
[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b^4\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{1}{9}\left(7b^2\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{7}{15}\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15bd\sqrt{b\cos(c+dx)}}-\frac{7\int \sqrt{b\cos(c+dx)}\,dx}{15b^2}\\ &= \frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15bd\sqrt{b\cos(c+dx)}}-\frac{\left(7\sqrt{b\cos(c+dx)}\right)\int \sqrt{\text{c}}}{15b^2\sqrt{\cos(c+}}\\ &= -\frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin}{15bd\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.181281, size = 80, normalized size = 0.63

$$\frac{42\sin(c+dx)-42\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\tan(c+dx)\sec(c+dx)\left(5\sec^2(c+dx)+7\right)}{45bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-42*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 2*Sec
[c + d*x]*(7 + 5*Sec[c + d*x]^2)*Tan[c + d*x])/(45*b*d*Sqrt[b*Cos[c + d*x]]
)
```

Maple [B] time = 3.647, size = 414, normalized size = 3.3

$$-2\,\frac{\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}}{b\sin(1/2\,dx+c/2)\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)}d}\left(-\frac{\cos(1/2\,dx+c/2)\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+\right.}}{144\,b\left((\cos(1/2\,dx+c/2))^2-1/2\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-1/144*cos(
1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1
/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*
d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\sec(dx+c)^4}{b^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^4/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

3.127 $\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^7d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^5d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77b^3d}+\frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{77b^2d\sqrt{b\cos(c}}$$

```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*b^2*d*Sqrt[b*Cos[c +
d*x]]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*b^3*d) + (18*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(77*b^5*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d
*x])/(11*b^7*d)
```

Rubi [A] time = 0.0823867, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^7d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^5d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77b^3d}+\frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{77b^2d\sqrt{b\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*b^2*d*Sqrt[b*Cos[c +
d*x]]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*b^3*d) + (18*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(77*b^5*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d
*x])/(11*b^7*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^8}$$
$$= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^7d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b^6}$$
$$= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^5d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^7d} + \frac{45\int (b\cos(c+dx))^{3/2}\,d}{77b^4}$$
$$= \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77b^3d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^5d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c}{11b^7d}$$
$$= \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77b^3d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^5d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c}{11b^7d}$$
$$= \frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77b^2d\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77b^3d} + \frac{18(b\cos(c+dx))^{5/2}\sin(}{77b^5d}$$

Mathematica [A] time = 0.113657, size = 76, normalized size = 0.59

$$\frac{347\sin(2(c+dx)) + 64\sin(4(c+dx)) + 7\sin(6(c+dx)) + 480\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{1232b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (480*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 347*Sin[2*(c + d*x)] +
64*Sin[4*(c + d*x)] + 7*Sin[6*(c + d*x)])/(1232*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.044, size = 236, normalized size = 1.8

$$-\frac{2}{77\,b^2d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(448\,(\cos(1/2\,dx+c/2))^{13}-1568\,(\cos(1/2\,dx+c/2))^{11}+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(448*co
s(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-20
40*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+
15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*
c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*
c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^8}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^8/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{5}}{b^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^5/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{8}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^8/(b*cos(d*x + c))^(5/2), x)
```

3.128 $\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^6d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^4d}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}}$$

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sqrt[Cos[c +
d*x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^4*d) + (2*(b*Cos[c
+ d*x])^(7/2)*Sin[c + d*x])/(9*b^6*d)
```

Rubi [A] time = 0.0599299, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2640, 2639}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^6d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^4d}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sqrt[Cos[c +
d*x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^4*d) + (2*(b*Cos[c
+ d*x])^(7/2)*Sin[c + d*x])/(9*b^6*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^7}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{7\int (b\cos(c+dx))^{5/2}\,dx}{9b^5}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{7\int \sqrt{b\cos(c+dx)}\,dx}{15b^3}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{\left(7\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos}}{15b^3\sqrt{\cos(c+dx}}$$
$$= \frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2(b\cos(c+dx))^{7/2}\sin(}{9b^6d}$$

Mathematica [A] time = 0.0977524, size = 74, normalized size = 0.74

$$\frac{(38\sin(2(c+dx))+5\sin(4(c+dx)))\cos(c+dx)+168\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{180b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (168*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*(38*Sin[2*
(c + d*x)] + 5*Sin[4*(c + d*x)]))/(180*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.151, size = 223, normalized size = 2.2

$$-\frac{2}{45\,b^2d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(160\,(\cos(1/2\,dx+c/2))^{11}-480\,(\cos(1/2\,dx+c/2))^9+616\,(\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(160*co
s(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*c
os(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2
4*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^7}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\cos(dx+c)^4}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^4/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^7}{(b\cos(dx+c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(5/2), x)
```

3.129 $\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (2*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)
```

Rubi [A] time = 0.0601368, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(b*Cos[c + d*x])^(5/2),x]
```



```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (2*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b^6}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{5\int (b\cos(c+dx))^{3/2}\,dx}{7b^4}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{5\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b^2}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{\left(5\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}}}{21b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c}{7b^5d}$$

Mathematica [A] time = 0.0535276, size = 66, normalized size = 0.66

$$\frac{26\sin(2(c+dx)) + 3\sin(4(c+dx)) + 40\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{84b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26*Sin[2*(c + d*x)] + 3*
Sin[4*(c + d*x)])/(84*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.086, size = 210, normalized size = 2.1

$$-\frac{2}{21\,b^2d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(48\,(\cos(1/2\,dx+c/2))^9-120\,(\cos(1/2\,dx+c/2))^7+128\,(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(48*cos
(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(
1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*
cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\cos(dx+c)^3}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^3/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(5/2), x)
```

3.130 $\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}$$

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*
x]]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)
```

Rubi [A] time = 0.0402655, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2640, 2639}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*
x]]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{5/2}\,dx}{b^5}$$
$$= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b^3}$$
$$= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}}$$
$$= \frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}$$

Mathematica [A] time = 0.0454378, size = 61, normalized size = 0.85

$$\frac{\sin(2(c+dx))\cos(c+dx)+6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*Sin[2*(c + d
*x)])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 2.027, size = 213, normalized size = 3.

$$-\frac{2}{5\,b^2d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)+8\,(\sin(1/2\,dx+c/2)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-8*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\cos(dx+c)^2}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)^2/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

3.131 $\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```

Rubi [A] time = 0.0407445, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2635, 2642, 2641}

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\,dx}{b^4}$$
$$= \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}$$
$$= \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}$$

Mathematica [A] time = 0.0427678, size = 54, normalized size = 0.75

$$\frac{\sin(2(c+dx)) + 2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])/(3*b^2*
d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 1.984, size = 190, normalized size = 2.6

$$-\frac{2}{3\,b^2d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,(\sin(1/2\,dx+c/2))^4\cos(1/2\,dx+c/2)+\sqrt{2\,(\sin(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(4*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\cos(dx+c)}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*cos(d*x + c)/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

3.132 $\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]
])
```

Rubi [A] time = 0.0228908, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 2640, 2639}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\int \sqrt{b\cos(c+dx)}\,dx}{b^3}\\ &= \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b^3\sqrt{\cos(c+dx)}}\\ &= \frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0146561, size = 41, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]
])
```

Maple [B] time = 1.608, size = 144, normalized size = 3.5

$$2\,\frac{\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\mathrm{EllipticE}\left(\cos\right.}{b^2\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+c/2))^2\right)}\sin(1/2\,dx+c/2)\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/b^2/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)}}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

3.133 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Cos[c + d*x]
])
```

Rubi [A] time = 0.0255456, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 2642, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}\\
&= \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.059427, size = 38, normalized size = 0.93

$$\frac{2\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2])/(d*(b*Cos[c + d*x])^(5/2))
```

Maple [B] time = 1.586, size = 144, normalized size = 3.5

$$-2\frac{\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/2\,dx+c/2)\right)^2+1}\,\mathrm{EllipticF}\left(\cdots\right.}{b^2\sqrt{-b\left(2\left(\sin(1/2\,dx+c/2)\right)^4-\left(\sin(1/2\,dx+c/2)\right)^2\right)}\sin(1/2\,dx+c/2)\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/si
n(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^2}{(b\cos (dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

3.134 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x
]]) + (2*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0422722, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 2636, 2640, 2639}

$$\frac{2\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x
]]) + (2*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{b}$$
$$= \frac{2\sin(c+dx)}{b^2 d\sqrt{b\cos(c+dx)}} - \frac{\int \sqrt{b\cos(c+dx)}\,dx}{b^3}$$
$$= \frac{2\sin(c+dx)}{b^2 d\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b^3\sqrt{\cos(c+dx)}}$$
$$= -\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3 d\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{b^2 d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0378036, size = 50, normalized size = 0.74

$$\frac{2\left(\sin(c+dx) - \sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{b^2 d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(b^2*d
*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.231, size = 168, normalized size = 2.5

$$-2\frac{\sqrt{-2b(\sin(1/2dx+c/2))^4+(\sin(1/2dx+c/2))^2 b}\left(\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{2(\sin(1/2dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos\right.\right.}{b^2\sqrt{-b\left(2(\sin(1/2dx+c/2))^4-(\sin(1/2dx+c/2))^2\right)}\sin(1/2dx+c/2)\sqrt{b\left(2(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*((sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*
x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.135 $\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0353592, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2642, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}\\ &= \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0177185, size = 51, normalized size = 0.71

$$\frac{2\left(\tan(c+dx)+\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*b^2*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 1.909, size = 241, normalized size = 3.4

$$-\frac{2}{3\,b^2d}\left(-2\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\text{EllipticF}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)\sqrt{(\sin(1/2\,dx+c/2))^2}\,(\sin(1/2\,dx+c/2))^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/b^2*(b*(2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/
2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(-5/2), x)
```

3.136 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d
*x]]) + (2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/(5
*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0696084, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 2636, 2640, 2639}

$$\frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d
*x]]) + (2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/(5
*b^2*d*Sqrt[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b}$$
$$= \frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} - \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b^3}$$
$$= \frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} - \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}}$$
$$= -\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0596633, size = 68, normalized size = 0.7

$$\frac{6\sin(c+dx) - 6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\tan(c+dx)\sec(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[c
 + d*x]*Tan[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 3.269, size = 366, normalized size = 3.8

$$\frac{2}{5\,b^3d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2*d
*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)}{b^{3}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)/(b^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.137 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}+\frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c +
d*x]]) + (2*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*Sin[c + d*x]
)/(21*b*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0792451, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2642, 2641}

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}+\frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c +
d*x]]) + (2*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*Sin[c + d*x]
)/(21*b*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b^2\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{5}{7}\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b^2}$$
$$= \frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{\left(5\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0709466, size = 66, normalized size = 0.67

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\tan(c+dx)\left(3\sec^2(c+dx)+5\right)}{21b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5 + 3*Sec[c + d*x]^2)
*Tan[c + d*x])/(21*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 1.937, size = 398, normalized size = 4.1

$$-\frac{2}{21\,b^2d}\left(-40\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,(\sin(1/2\,dx+c/2))\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-3
0*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c))/b^2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\sec(dx+c)^2}{b^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

3.138 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14\sin(c+dx)}{15b^2d\sqrt{b\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}$$

```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sqrt[Cos[c +
 d*x]]) + (2*b^2*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*Sin[c + d
*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*Sin[c + d*x])/(15*b^2*d*Sqrt[b*Cos
[c + d*x]])
```

Rubi [A] time = 0.0979237, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 2636, 2640, 2639}

$$\frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14\sin(c+dx)}{15b^2d\sqrt{b\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sqrt[Cos[c +
 d*x]]) + (2*b^2*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*Sin[c + d
*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*Sin[c + d*x])/(15*b^2*d*Sqrt[b*Cos
[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b^3\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}(7b)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{7\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{15b}$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15b^2d\sqrt{b\cos(c+dx)}} - \frac{7\int \sqrt{b\cos(c+dx)}\,dx}{15b^3}$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15b^2d\sqrt{b\cos(c+dx)}} - \frac{\left(7\sqrt{b\cos(c+dx)}\right)\int \sqrt{c}}{15b^3\sqrt{\cos(c+}}$$
$$= -\frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin}{15b^2d\sqrt{b}}$$

Mathematica [A] time = 0.088816, size = 80, normalized size = 0.64

$$\frac{42\sin(c+dx) - 42\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\tan(c+dx)\sec(c+dx)\left(5\sec^2(c+dx)+7\right)}{45b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-42*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 2*Sec
[c + d*x]*(7 + 5*Sec[c + d*x]^2)*Tan[c + d*x])/(45*b^2*d*Sqrt[b*Cos[c + d*x
]])
```

Maple [B] time = 3.252, size = 414, normalized size = 3.3

$$-2\,\frac{\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}}{b^2\sin(1/2\,dx+c/2)\,\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)}d}\left(-\frac{\cos(1/2\,dx+c/2)\,\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+\right.}}{144\,b\left((\cos(1/2\,dx+c/2))^2-1/2\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-1/144*co
s(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/
2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}\sec(dx+c)^3}{b^3\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))*sec(d*x + c)^3/(b^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

3.139 $\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c + d
*x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/
(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0531143, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2640, 2639}

$$\frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c + d
*x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/
(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b^2}$$
$$= \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b^4}$$
$$= \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^4\sqrt{\cos(c+dx)}}$$
$$= -\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0176344, size = 68, normalized size = 0.68

$$\frac{6\sin(c+dx) - 6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\tan(c+dx)\sec(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[c
 + d*x]*Tan[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 3.435, size = 366, normalized size = 3.7

$$\frac{2}{5\,b^4d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(7/2),x)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2*d
*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)}}{b^4\cos(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c))/(b^4*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(-7/2), x)
```

3.140 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{3x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}$$

```
[Out] (3*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (3*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0271258, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{3x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (3*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (3*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x])/(4*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int \cos^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$
$$= \frac{3\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$
$$= \frac{3x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{3\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\,\mathrm{co}}}{}$$

Mathematica [A] time = 0.0950277, size = 55, normalized size = 0.56

$$\frac{(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))\sqrt{b\cos(c+dx)}}{32d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
))/(32*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.316, size = 62, normalized size = 0.6

$$\frac{2\,(\cos(dx+c))^3\sin(dx+c)+3\,\cos(dx+c)\sin(dx+c)+3\,dx+3\,c}{8\,d}\sqrt{b\cos(dx+c)}\frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(b*cos(d*x+c))^(1/2)*(2*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+
c)+3*d*x+3*c)/cos(d*x+c)^(1/2)
```

Maxima [A] time = 1.81787, size = 66, normalized size = 0.67

$$\frac{\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))*sqrt(b)/d
```

Fricas [A] time = 1.93805, size = 495, normalized size = 5.05

$$\left[\frac{2\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+3\right)\sqrt{\cos(dx+c)}\sin(dx+c)+3\sqrt{-b}\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-\right.}{16d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin
(d*x + c) + 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt
(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/8*(sqrt(b*cos(d*x + c))*(2*
cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*arctan(sqrt
(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.141 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0172808, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{\sqrt{b\cos(c+dx)}\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\ &= \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0889298, size = 45, normalized size = 0.64

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)\sqrt{b\cos(c+dx)}}{6d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqrt[Cos[c
+ d*x]])
```

Maple [A] time = 0.283, size = 40, normalized size = 0.6

$$\frac{\left(2+\left(\cos\left(dx+c\right)\right)^{2}\right)\sin\left(dx+c\right)}{3\,d}\sqrt{b\cos\left(dx+c\right)}\frac{1}{\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*(2+cos(d*x+c)^2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [A] time = 1.82806, size = 57, normalized size = 0.81

$$\frac{\sqrt{b}\left(\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))))/d
```

Fricas [A] time = 1.70676, size = 112, normalized size = 1.6

$$\frac{\sqrt{b\cos\left(dx+c\right)}\left(\cos\left(dx+c\right)^{2}+2\right)\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(d*sqrt(cos(d*x
+ c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.142 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[Out] (x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[
b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0141595, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[
b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}+\frac{\sqrt{b\cos(c+dx)}\int 1\,dx}{2\sqrt{\cos(c+dx)}}\\
&= \frac{x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0546927, size = 45, normalized size = 0.71

$$\frac{(2(c+dx)+\sin(2(c+dx)))\sqrt{b\cos(c+dx)}}{4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[Cos[c + d
*x]])
```

Maple [A] time = 0.273, size = 42, normalized size = 0.7

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d}\sqrt{b\cos(dx+c)}\frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(1/2)*(cos(d*x+c)*sin(d*x+c)+d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [A] time = 1.85081, size = 34, normalized size = 0.54

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A] time = 1.89939, size = 427, normalized size = 6.78

$$\left[\frac{2\,\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{-b}\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)\right.}{4\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(-b)*log
(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*si
n(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)
^(3/2))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.143 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0067261, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0281434, size = 32, normalized size = 1.

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.269, size = 29, normalized size = 0.9

$$\frac{\sin (dx+c)}{d}\sqrt{b\cos (dx+c)}\frac{1}{\sqrt{\cos (dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Maxima [A] time = 1.7803, size = 18, normalized size = 0.56

$$\frac{\sqrt{b}\sin (dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b)*sin(d*x + c)/d
```

Fricas [A] time = 1.61199, size = 78, normalized size = 2.44

$$\frac{\sqrt{b\cos (dx+c)}\sin (dx+c)}{d\sqrt{\cos (dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [A] time = 20.8331, size = 29, normalized size = 0.91

$$\begin{cases} \frac{\sqrt{b}\sin (c+dx)}{d} & \text{for } d\neq 0 \\ x\sqrt{b\cos (c)}\sqrt{\cos (c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((sqrt(b)*sin(c + d*x)/d, Ne(d, 0)), (x*sqrt(b*cos(c))*sqrt(cos(c)
), True))
```

Giac [A] time = 3.37655, size = 42, normalized size = 1.31

$$\frac{2\sqrt{b}\tan \left(\frac{1}{2}dx+\frac{1}{2}c\right)}{d\tan \left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b)*tan(1/2*d*x + 1/2*c)/(d*tan(1/2*d*x + 1/2*c)^2 + d)
```

3.144 $\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] (x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Rubi [A] time = 0.0025662, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int 1\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0108263, size = 24, normalized size = 1.

$$\frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Maple [A] time = 0.188, size = 28, normalized size = 1.2

$$\frac{dx+c}{d}\sqrt{b\cos\left(dx+c\right)}\frac{1}{\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(d*x+c)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [A] time = 1.5425, size = 35, normalized size = 1.46

$$\frac{2\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A] time = 1.86571, size = 265, normalized size = 11.04

$$\left[\frac{\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^{2}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)}{2\,d}, \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt
(cos(d*x + c))*sin(d*x + c) - b)/d, sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin
(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/d]
```

Sympy [A] time = 3.87443, size = 5, normalized size = 0.21

$$\sqrt{b}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] sqrt(b)*x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \cos (dx + c)}}{\sqrt{\cos (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

3.145 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0077919, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0123677, size = 33, normalized size = 1.

$$\frac{\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.268, size = 42, normalized size = 1.3

$$-2\frac{\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}\text{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1
/2)
```

Maxima [B] time = 1.82238, size = 88, normalized size = 2.67

$$\frac{\sqrt{b}\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.84106, size = 311, normalized size = 9.42

$$\left[\frac{\sqrt{b}\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/d, -sqrt(-b)*
arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))/d
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

3.146 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.0118561, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3767, 8}

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]

[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{\sqrt{b\cos(c+dx)}\,\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.0212212, size = 32, normalized size = 1.

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.271, size = 29, normalized size = 0.9

$$\frac{\sin (dx+c)}{d}\sqrt{b\cos (dx+c)}\,(\cos (dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Maxima [A] time = 1.76541, size = 73, normalized size = 2.28

$$\frac{2\sqrt{b}\sin (2\,dx+2\,c)}{\left(\cos (2\,dx+2\,c)^{2}+\sin (2\,dx+2\,c)^{2}+2\,\cos (2\,dx+2\,c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.62541, size = 78, normalized size = 2.44

$$\frac{\sqrt{b\cos (dx+c)}\sin (dx+c)}{d\cos (dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

3.147 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.0206031, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{\sqrt{b\cos(c+dx)}\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}\\
&= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.0473009, size = 52, normalized size = 0.72

$$\frac{\sqrt{b\cos(c+dx)}\left(\sin(c+dx)+\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x])
)/(2*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.289, size = 104, normalized size = 1.4

$$-\frac{1}{2d}\left((\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-(\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))*(b*cos(d*x+c))^(1/
2)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 1.83752, size = 892, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
 2*c) + 1)*d)
```

Fricas [A] time = 1.88491, size = 559, normalized size = 7.76

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+\right.}{4\,d\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c
)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

3.148 $\int \frac{\sqrt{b \cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)} \, dx$

Optimal. Leaf size=70

$$\frac{\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.018236, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3767}

$$\frac{\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{\sqrt{b\cos(c+dx)}\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\ &= \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.0845274, size = 45, normalized size = 0.64

$$\frac{\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sqrt[Cos[c + d*
x]])
```

Maple [A] time = 0.269, size = 42, normalized size = 0.6

$$\frac{\left(2\,(\cos(dx+c))^2+1\right)\sin(dx+c)}{3\,d}\sqrt{b\cos(dx+c)}\,(\cos(dx+c))^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 1.8401, size = 397, normalized size = 5.67

$$\frac{4\,((3\,\cos(2\,dx+2\,c)+1)\,\mathrm{si}}{3\left(2\,(3\,\cos(4\,dx+4\,c)+3\,\cos(2\,dx+2\,c)+1)\cos(6\,dx+6\,c)+\cos(6\,dx+6\,c)^2+6\,(3\,\cos(2\,dx+2\,c)+1)\cos(4\,dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) +
1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x +
4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*
x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 +
6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.63931, size = 115, normalized size = 1.64

$$\frac{\sqrt{b\cos(dx+c)}\left(2\,\cos(dx+c)^2+1\right)\sin(dx+c)}{3\,d\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(d*cos(d*x + c
)^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

3.149 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{3\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (3*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.0359618, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{3\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(11/2),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (3*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int \sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$
$$= \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)}{8\sqrt{\cos(c+dx)}}$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3\sqrt{b\cos(c+dx)}\sin(c+}{8d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] time = 0.106075, size = 66, normalized size = 0.62

$$\frac{\sqrt{b\cos(c+dx)}\left(\sin(c+dx)\left(3\cos^2(c+dx)+2\right)+3\cos^4(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{8d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(11/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[
c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.306, size = 121, normalized size = 1.1

$$-\frac{1}{8d}\left(3\left(\cos(dx+c)\right)^4\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-3\left(\cos(dx+c)\right)^4\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-3*cos(d*x
+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+c)^2*sin(d*x+c)-
2*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 2.07954, size = 2236, normalized size = 20.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*s
in(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*
(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x +
 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
 + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
```

```
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^
2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*c
os(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4
*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6
*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*s
in(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(
2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4
*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x
+ 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2
 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d
*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.92371, size = 628, normalized size = 5.87

$$\left[\frac{3\sqrt{b}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^{2}+2\right)}{16\,d\cos\left(dx+c\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(d*x + c)
)*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - sqrt(b*cos
(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
```

```
*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \cos(dx+c)}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(11/2), x)
```

3.150 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=101

$$\frac{3bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3b\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}$$

```
[Out] (3*b*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (3*b*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*Cos[c + d*x]^(5/2)*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0286958, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{3bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3b\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (3*b*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (3*b*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*Cos[c + d*x]^(5/2)*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{b\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left(3b\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)}{4\sqrt{\cos(c+dx)}}$$
$$= \frac{3b\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin}{4d}$$
$$= \frac{3bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{3b\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b\cos^{\frac{5}{2}}(c+}{}$$

Mathematica [A] time = 0.0885555, size = 55, normalized size = 0.54

$$\frac{(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))(b\cos(c+dx))^{3/2}}{32d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x
)]))/(32*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.184, size = 62, normalized size = 0.6

$$\frac{2\,(\cos(dx+c))^3\sin(dx+c)+3\,\cos(dx+c)\sin(dx+c)+3\,dx+3\,c}{8\,d}(b\cos(dx+c))^{\frac{3}{2}}(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/8/d*(b*cos(d*x+c))^(3/2)*(2*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+
c)+3*d*x+3*c)/cos(d*x+c)^(3/2)
```

Maxima [A] time = 1.82659, size = 72, normalized size = 0.71

$$\frac{\left(12\,(dx+c)b+b\sin(4\,dx+4\,c)+8\,b\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))))*sqrt(b)/d
```

Fricas [A] time = 1.93303, size = 509, normalized size = 5.04

$$\left[\frac{2\left(2\,b\cos\left(dx+c\right)^{2}+3\,b\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\,\sqrt{-b}b\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{\cdot}\right.}{16\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*b*cos(d*x + c)^2 + 3*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) + 3*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c)
)*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/8*((2*b*cos(d*x + c)^
2 + 3*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*b^(3/2)*a
rctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.151 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0182427, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.11678, size = 45, normalized size = 0.62

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)(b\cos(c+dx))^{3/2}}{6d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Cos[c + d
*x]^(3/2))
```

Maple [A] time = 0.175, size = 40, normalized size = 0.6

$$\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3d}(b\cos(dx+c))^{\frac{3}{2}}(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/3/d*(2+cos(d*x+c)^2)*sin(d*x+c)*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2)
```

Maxima [A] time = 1.81457, size = 61, normalized size = 0.85

$$\frac{\left(b\sin(3\,dx+3\,c)+9\,b\sin\left(\frac{1}{3}\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))))*sqrt(b)/d
```

Fricas [A] time = 1.67918, size = 117, normalized size = 1.62

$$\frac{\left(b\cos(dx+c)^2+2\,b\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{3\,d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*cos(d*x + c)^2 + 2*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(
d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.152 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[Out] (b*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.014842, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (b*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}+\frac{\left(b\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{\cos(c+dx)}}\\ &= \frac{bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0545278, size = 45, normalized size = 0.69

$$\frac{(2(c+dx)+\sin(2(c+dx)))(b\cos(c+dx))^{3/2}}{4d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Cos[c + d*x]
^(3/2))
```

Maple [A] time = 0.25, size = 42, normalized size = 0.7

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d}(b\cos(dx+c))^{\frac{3}{2}}(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(3/2)*(cos(d*x+c)*sin(d*x+c)+d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [A] time = 1.78711, size = 38, normalized size = 0.58

$$\frac{(2(dx+c)b+b\sin(2dx+2c))\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A] time = 1.95001, size = 435, normalized size = 6.69

$$\left[\frac{2\sqrt{b\cos(dx+c)}b\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{-b}b\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(\right.}{4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*b*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(-b)*b
*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c)
)*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*b*sqrt(cos(d*x + c))*sin(
d*x + c) + b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.153 $\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=33

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0069987, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0441519, size = 32, normalized size = 0.97

$$\frac{\sin(c+dx)(b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.224, size = 29, normalized size = 0.9

$$\frac{\sin\left(dx+c\right)}{d}\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(\cos\left(dx+c\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [A] time = 1.77032, size = 18, normalized size = 0.55

$$\frac{b^{\frac{3}{2}}\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] b^(3/2)*sin(d*x + c)/d
```

Fricas [A] time = 1.58931, size = 81, normalized size = 2.45

$$\frac{\sqrt{b\cos\left(dx+c\right)}b\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*b*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)), x)
```

3.154 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=25

$$\frac{bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] (b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Rubi [A] time = 0.0027328, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0159376, size = 24, normalized size = 0.96

$$\frac{x(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (x*(b*Cos[c + d*x])^(3/2))/Cos[c + d*x]^(3/2)
```

Maple [A] time = 0.145, size = 28, normalized size = 1.1

$$\frac{dx+c}{d}(b\cos(dx+c))^{\frac{3}{2}}(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(d*x+c)*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2)
```

Maxima [A] time = 1.54437, size = 35, normalized size = 1.4

$$\frac{2\,b^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A] time = 1.87889, size = 267, normalized size = 10.68

$$\left[\frac{\sqrt{-b}b\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)}{2\,d},\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sq
rt(cos(d*x + c))*sin(d*x + c) - b)/d, b^(3/2)*arctan(sqrt(b*cos(d*x + c))*s
in(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2), x)
```

3.155 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{b\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.007926, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{b\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0156557, size = 33, normalized size = 0.97

$$\frac{(b\cos(c+dx))^{3/2}\tanh^{-1}(\sin(c+dx))}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*(b*Cos[c + d*x])^(3/2))/(d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.162, size = 42, normalized size = 1.2

$$-2\,{\frac {(b\cos (dx+c))^{3/2}}{d(\cos (dx+c))^{3/2}}}\text{Artanh}\left({\frac {-1+\cos (dx+c)}{\sin (dx+c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3
/2)
```

Maxima [B] time = 1.82723, size = 92, normalized size = 2.71

$$\frac{\left(b\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-b\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)\right)\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

Fricas [A] time = 1.93624, size = 313, normalized size = 9.21

$$\left[\frac{b^{\frac{3}{2}}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)}{2\,d},\,-\frac{\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*b^(3/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/d, -sqrt(-b)*
b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))
/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2), x)
```

3.156 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.012162, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3767, 8}

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.0234374, size = 32, normalized size = 0.97

$$\frac{\sin(c+dx)(b\cos(c+dx))^{3/2}}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2))
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x)
```

Maxima [A] time = 1.80648, size = 73, normalized size = 2.21

$$\frac{2\,b^{\frac{3}{2}}\sin\left(2\,dx+2\,c\right)}{\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2*b^(3/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.61215, size = 81, normalized size = 2.45

$$\frac{\sqrt{b\cos\left(dx+c\right)}b\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*b*sin(d*x + c)/(d*cos(d*x + c)^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(7/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2), x)
```

3.157 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.0210489, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.0535812, size = 52, normalized size = 0.7

$$\frac{(b\cos(c+dx))^{3/2}\left(\sin(c+dx)+\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x
]))/(2*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.17, size = 104, normalized size = 1.4

$$-\frac{1}{2d}\left((\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-(\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))*(b*cos(d*x+c))^(3/
2)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 1.88876, size = 933, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*
c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*c
os(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (
b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d
*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*co
s(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*s
qrt(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.93185, size = 567, normalized size = 7.66

$$\left[\frac{b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}b\sqrt{\cos\left(dx+c\right)}\sin\left(\right.}{4\,d\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^3), -1/2*(sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*b*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(9/2), x)
```

3.158 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{b\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.0183373, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3767}

$$\frac{b\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\ &= \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.0604555, size = 45, normalized size = 0.62

$$\frac{\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)(b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Cos[c + d*x]^
(3/2))
```

Maple [A] time = 0.165, size = 42, normalized size = 0.6

$$\frac{\left(2\left(\cos\left(dx+c\right)\right)^{2}+1\right)\sin\left(dx+c\right)}{3d}\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(\cos\left(dx+c\right)\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 1.82876, size = 404, normalized size = 5.61

$$-\frac{4\left(3\,b\cos\left(6\,dx+6\,c\right)\sin\left(\cdots\right.\right.}{3\left(2\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^{2}+6\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -4/3*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*x + 4*c)*sin(2*d*
x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3*(3*b*cos(2*d*x +
 2*c) + b)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin
(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9
*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x +
2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.60906, size = 117, normalized size = 1.62

$$\frac{\left(2\,b\cos\left(dx+c\right)^{2}+b\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*cos(d*x + c)^2 + b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
 c)^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(11/2), x)
```

3.159 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{3b\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3b\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}$$

```
[Out] (3*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) +
 (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (3*b*Sqrt
[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.0392478, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{3b\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3b\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) +
 (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (3*b*Sqrt
[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(3b\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$
$$= \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3b\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(3b\sqrt{b\cos(c+dx)}\right)\int \sec(c+d}{8\sqrt{\cos(c+dx)}}$$
$$= \frac{3b\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3b\sqrt{b\cos(c+dx)}\sin(}{8d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] time = 0.0887455, size = 67, normalized size = 0.61

$$\frac{b\sqrt{b\cos(c+dx)}\left(\sin(c+dx)\left(3\cos^2(c+dx)+2\right)+3\cos^4(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{8d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Co
s[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.195, size = 121, normalized size = 1.1

$$-\frac{1}{8d}\left(3(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-3(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-3*cos(d*x
+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+c)^2*sin(d*x+c)-
2*sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2)
```

Maxima [B] time = 2.19676, size = 2352, normalized size = 21.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c)
 + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) +
 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4
```

```
*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*b
*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3
*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2
+ 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2
+ 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*s
in(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*
d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
+ 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4
*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2
*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*
(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2 +
 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2 +
 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*si
n(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d
*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x +
 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4*
c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2*
b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*
(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4
4*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*c
os(2*d*x + 2*c) + b)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b
*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4
*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*
c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*c
os(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x +
2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
 + 2*c) + 1)*d)
```

Fricas [A] time = 2.15479, size = 641, normalized size = 5.83

$$\left[\frac{3\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(3\,b\cos\left(dx+c\right)^{2}+2\,b\right)\sqrt{b\cos\left(dx\right.}}{16\,d\cos\left(dx+c\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*(3*b*cos(d*x + c)^2 + 2*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*b*arctan(sqrt(b*cos(d*
```

```
x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (3*b
*cos(d*x + c)^2 + 2*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(13/2), x)
```

3.160 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{b^2\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (2*b^2*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (b^2*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.025976, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{b^2\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (2*b^2*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (b^2*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\ &= \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2b^2\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{b^2\sqrt{b\cos(c+}}{5d\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.125421, size = 57, normalized size = 0.49

$$\frac{\sin(c+dx)\left(3\sin^4(c+dx)-10\sin^2(c+dx)+15\right)(b\cos(c+dx))^{5/2}}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sin[c + d*x]*(15 - 10*Sin[c + d*x]^2 + 3*Sin[c + d*
x]^4))/(15*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.169, size = 52, normalized size = 0.5

$$\frac{\left(3 \left(\cos \left(dx+c\right)\right)^{4}+4 \left(\cos \left(dx+c\right)\right)^{2}+8\right) \sin \left(dx+c\right)}{15 d} \left(b \cos \left(dx+c\right)\right)^{\frac{5}{2}} \left(\cos \left(dx+c\right)\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/15/d*(3*cos(d*x+c)^4+4*cos(d*x+c)^2+8)*sin(d*x+c)*(b*cos(d*x+c))^(5/2)/co
s(d*x+c)^(5/2)
```

Maxima [A] time = 1.86682, size = 104, normalized size = 0.9

$$\frac{\left(3 b^{2} \sin \left(5 dx+5 c\right)+25 b^{2} \sin \left(\frac{3}{5} \arctan \left(\sin \left(5 dx+5 c\right), \cos \left(5 dx+5 c\right)\right)\right)+150 b^{2} \sin \left(\frac{1}{5} \arctan \left(\sin \left(5 dx+5 c\right), \mathrm{cc}\right.\right.}{240 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*arctan2(sin(5*d*x + 5*c), co
s(5*d*x + 5*c))) + 150*b^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c))))*sqrt(b)/d
```

Fricas [A] time = 1.97581, size = 158, normalized size = 1.36

$$\frac{\left(3 b^{2} \cos \left(dx+c\right)^{4}+4 b^{2} \cos \left(dx+c\right)^{2}+8 b^{2}\right) \sqrt{b \cos \left(dx+c\right)} \sin \left(dx+c\right)}{15 d \sqrt{\cos \left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*cos(d*x + c)^4 + 4*b^2*cos(d*x + c)^2 + 8*b^2)*sqrt(b*cos(d*x +
 c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.161 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{3b^2x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}$$

```
[Out] (3*b^2*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (3*b^2*Sqrt[Cos[c +
 d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]^(5/2)*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0299723, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{3b^2x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (3*b^2*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (3*b^2*Sqrt[Cos[c +
 d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]^(5/2)*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left(3b^2\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+d}{4\sqrt{\cos(c+dx)}}$$

$$= \frac{3b^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b^2\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\mathrm{s}}{4d}$$

$$= \frac{3b^2x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{3b^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b^2\cos^{\frac{5}{2}}(c}{}$$

Mathematica [A] time = 0.0845613, size = 55, normalized size = 0.51

$$\frac{(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))(b\cos(c+dx))^{5/2}}{32d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x
)]))/(32*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.175, size = 62, normalized size = 0.6

$$\frac{2\,(\cos(dx+c))^3\sin(dx+c)+3\,\cos(dx+c)\sin(dx+c)+3\,dx+3\,c}{8\,d}(b\cos(dx+c))^{\frac{5}{2}}(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/8/d*(b*cos(d*x+c))^(5/2)*(2*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+
c)+3*d*x+3*c)/cos(d*x+c)^(5/2)
```

Maxima [A] time = 1.83253, size = 80, normalized size = 0.75

$$\frac{\left(12\,(dx+c)b^2+b^2\sin(4\,dx+4\,c)+8\,b^2\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))))*sqrt(b)/d
```

Fricas [A] time = 2.29515, size = 522, normalized size = 4.88

$$\left[\frac{3\,\sqrt{-b}b^2\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2\left(2\,b^2\cos\left(dx+c\right)^2+3\,b^2\right)\sqrt{b\cos\left(dx+c\right)}\right.}{16\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(
-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*b^2*cos(d*x + c)^2 + 3*b^2)
*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d, 1/8*(3*b^(5/2)*ar
ctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + (2*b
^2*cos(d*x + c)^2 + 3*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.162 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b^2*Sqrt[
b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0181426, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b^2*Sqrt[
b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b^2\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.143718, size = 45, normalized size = 0.59

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)(b\cos(c+dx))^{5/2}}{6d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Cos[c + d
*x]^(5/2))
```

Maple [A] time = 0.243, size = 40, normalized size = 0.5

$$\frac{\left(2+\left(\cos\left(dx+c\right)\right)^{2}\right)\sin\left(dx+c\right)}{3\,d}\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(\cos\left(dx+c\right)\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*(2+cos(d*x+c)^2)*sin(d*x+c)*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2)
```

Maxima [A] time = 1.81574, size = 66, normalized size = 0.87

$$\frac{\left(b^{2}\sin\left(3\,dx+3\,c\right)+9\,b^{2}\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))))*sqrt(b)/d
```

Fricas [A] time = 1.98696, size = 123, normalized size = 1.62

$$\frac{\left(b^{2}\cos\left(dx+c\right)^{2}+2\,b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*cos(d*x + c)^2 + 2*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(
cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.163 $\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=69

$$\frac{b^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[Out] (b^2*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0155617, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{b^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b^2*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}+\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{\cos(c+dx)}}\\ &= \frac{b^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0723163, size = 45, normalized size = 0.65

$$\frac{(2(c+dx)+\sin(2(c+dx)))(b\cos(c+dx))^{5/2}}{4d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Cos[c + d*x]
^(5/2))
```

Maple [A] time = 0.234, size = 42, normalized size = 0.6

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d}(b\cos(dx+c))^{\frac{5}{2}}(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(cos(d*x+c)*sin(d*x+c)+d*x+c)*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2)
```

Maxima [A] time = 1.82901, size = 43, normalized size = 0.62

$$\frac{\left(2(dx+c)b^2+b^2\sin(2dx+2c)\right)\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A] time = 2.19279, size = 443, normalized size = 6.42

$$\left[\frac{2\sqrt{b\cos(dx+c)}b^2\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{-b}b^2\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin\right.}{4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(-b)
*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x
+ c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*b^2*sqrt(cos(d*x + c)
)*sin(d*x + c) + b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*
cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)), x)
```

3.164 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.008181, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0526622, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx)(b\cos(c+dx))^{5/2}}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.154, size = 29, normalized size = 0.8

$$\frac{\sin(dx+c)}{d}(b\cos(dx+c))^{\frac{5}{2}}(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [A] time = 1.77042, size = 18, normalized size = 0.51

$$\frac{b^{\frac{5}{2}}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b^(5/2)*sin(d*x + c)/d
```

Fricas [A] time = 1.86438, size = 84, normalized size = 2.4

$$\frac{\sqrt{b\cos(dx+c)}b^2\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*b^2*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2), x)
```

3.165 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{b^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Rubi [A] time = 0.0027092, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{b^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0123551, size = 24, normalized size = 0.89

$$\frac{x(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (x*(b*Cos[c + d*x])^(5/2))/Cos[c + d*x]^(5/2)
```

Maple [A] time = 0.131, size = 28, normalized size = 1.

$$\frac{dx+c}{d}(b\cos(dx+c))^{\frac{5}{2}}(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/d*(d*x+c)*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2)
```

Maxima [A] time = 1.53812, size = 35, normalized size = 1.3

$$\frac{2\,b^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A] time = 2.15123, size = 270, normalized size = 10.

$$\left[\frac{\sqrt{-b}b^2\log\left(2\,b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)}{2\,d},\ \frac{b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*
sqrt(cos(d*x + c))*sin(d*x + c) - b)/d, b^(5/2)*arctan(sqrt(b*cos(d*x + c))
*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2), x)
```

3.166 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{b^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0094418, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{b^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0221454, size = 33, normalized size = 0.92

$$\frac{(b\cos(c+dx))^{5/2}\tanh^{-1}(\sin(c+dx))}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*(b*Cos[c + d*x])^(5/2))/(d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.145, size = 42, normalized size = 1.2

$$-2\,\frac{(b\cos(dx+c))^{5/2}}{d\,(\cos(dx+c))^{5/2}}\,\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5
/2)
```

Maxima [B] time = 1.8455, size = 97, normalized size = 2.69

$$\frac{\left(b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

Fricas [A] time = 2.12115, size = 316, normalized size = 8.78

$$\left[\frac{b^{\frac{5}{2}}\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2\,d},\,-\frac{\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/2*b^(5/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/d, -sqrt(-b)*
b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))
))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(7/2), x)
```

3.167 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.0123448, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3767, 8}

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.0195227, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx)(b\cos(c+dx))^{5/2}}{d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.154, size = 29, normalized size = 0.8

$$\frac{\sin(dx+c)}{d}(b\cos(dx+c))^{\frac{5}{2}}(\cos(dx+c))^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] 1/d*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [A] time = 1.79481, size = 73, normalized size = 2.09

$$\frac{2\,b^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^{2}+\sin(2\,dx+2\,c)^{2}+2\,\cos(2\,dx+2\,c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2*b^(5/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.93341, size = 84, normalized size = 2.4

$$\frac{\sqrt{b\cos(dx+c)}b^{2}\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*b^2*sin(d*x + c)/(d*cos(d*x + c)^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(9/2), x)
```

3.168 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.0212637, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx&=\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\&=\frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}\\&=\frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.0597304, size = 52, normalized size = 0.67

$$\frac{(b\cos(c+dx))^{5/2}\left(\sin(c+dx)+\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x
]))/(2*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.162, size = 104, normalized size = 1.3

$$-\frac{1}{2d}\left((\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-(\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))*(b*cos(d*x+c))^(5/
2)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 1.85142, size = 1008, normalized size = 12.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d
*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(
4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*
d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2
 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2
*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2
*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x
 + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 2.1825, size = 575, normalized size = 7.37

$$\left[\frac{b^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}b^{2}\sqrt{\cos\left(dx+c\right)}\sin}{4\,d\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^3), -1/2*(sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x +
 c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*b^2*sqrt(
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2), x)
```

3.169 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b^2*Sqrt[
b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.0191562, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3767}

$$\frac{b^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b^2*Sqrt[
b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int\left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\ &= \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.0594675, size = 45, normalized size = 0.59

$$\frac{\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)(b\cos(c+dx))^{5/2}}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Cos[c + d*x]^
(5/2))
```

Maple [A] time = 0.159, size = 42, normalized size = 0.6

$$\frac{\left(2\,(\cos(dx+c))^2+1\right)\sin(dx+c)}{3d}(b\cos(dx+c))^{\frac{5}{2}}(\cos(dx+c))^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/cos(d*x+c)^(11/2)
```

Maxima [B] time = 1.87977, size = 420, normalized size = 5.53

$$-\frac{4\left(3\,b^2\cos(6\,dx+6\,c)\sin(2 \ldots\right.}{3\left(2\,(3\cos(4\,dx+4\,c)+3\cos(2\,dx+2\,c)+1)\cos(6\,dx+6\,c)+\cos(6\,dx+6\,c)^2+6\,(3\cos(2\,dx+2\,c)+1)\cos(4 \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="maxima")
```

```
[Out] -4/3*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c) - 3*(3*b^2*c
os(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) +
3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c
)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x
+ 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*
sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.85184, size = 123, normalized size = 1.62

$$\frac{\left(2\,b^2\cos(dx+c)^2+b^2\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{3\,d\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*cos(d*x + c)^2 + b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d
*x + c)^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(13/2), x)
```

3.170 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{3b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3b^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}$$

```
[Out] (3*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]])
 + (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (3*b^
2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.0358522, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{3b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3b^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(15/2),x]
```

```
[Out] (3*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]])
 + (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (3*b^
2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(3b^2\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(3b^2\sqrt{b\cos(c+dx)}\right)\int \sec(c}{8\sqrt{\cos(c+dx)}}$$

$$= \frac{3b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3b^2\sqrt{b\cos(c+dx)}\,s}{8d\cos^{\frac{5}{2}}(c+}$$

Mathematica [A] time = 0.115548, size = 66, normalized size = 0.57

$$\frac{(b\cos(c+dx))^{5/2}\left(\sin(c+dx)\left(3\cos^2(c+dx)+2\right)+3\cos^4(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{8d\cos^{\frac{13}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(15/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Co
s[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(13/2))
```

Maple [A] time = 0.192, size = 121, normalized size = 1.

$$\frac{1}{8d}\left(3(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-3(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x)
```

```
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+
c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2
*sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2)
```

Maxima [B] time = 2.21866, size = 2584, normalized size = 22.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d*x
+ 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(
4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^
```

```
2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c)
+ 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x
 + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin
(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 4
8*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2
*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c
) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4
*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x
+ 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*
x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x +
 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
1) + 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*d
*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*s
in(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 +
2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c)
 + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*
c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x +
4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
 + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(b^2*cos(8*d*x + 8
*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2
*c) + b^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(b^2*c
os(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*c
os(2*d*x + 2*c) + b^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 44*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*
c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 12*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*co
s(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*
cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x
+ 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*
c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) +
 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 3
6*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x
+ 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 2.19802, size = 655, normalized size = 5.65

$$\left[\frac{3\,b^{\frac{5}{2}}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(3\,b^{2}\cos\left(dx+c\right)^{2}+2\,b^{2}\right)\sqrt{b\cos\left(\right.}}{16\,d\cos\left(dx+c\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
```

```
c)^3) + 2*(3*b^2*cos(d*x + c)^2 + 2*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*b^2*arctan(sqrt(b*
cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5
- (3*b^2*cos(d*x + c)^2 + 2*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(15/2), x)
```

3.171 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=107

$$\frac{\sin^5(c+dx)\sqrt{\cos(c+dx)}}{5d\sqrt{b\cos(c+dx)}}-\frac{2\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) - (2*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Si
n[c + d*x]^5)/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0239209, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{\sin^5(c+dx)\sqrt{\cos(c+dx)}}{5d\sqrt{b\cos(c+dx)}}-\frac{2\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) - (2*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Si
n[c + d*x]^5)/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos^5(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}\\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.114257, size = 57, normalized size = 0.53

$$\frac{\sin(c+dx)\left(3\sin^4(c+dx)-10\sin^2(c+dx)+15\right)\sqrt{\cos(c+dx)}}{15d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x]*(15 - 10*Sin[c + d*x]^2 + 3*Sin[c + d*x]^4
))/(15*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.247, size = 52, normalized size = 0.5

$$\frac{\left(3\,(\cos(dx+c))^4+4\,(\cos(dx+c))^2+8\right)\sin(dx+c)}{15\,d}\sqrt{\cos(dx+c)}\frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d*(3*cos(d*x+c)^4+4*cos(d*x+c)^2+8)*sin(d*x+c)*cos(d*x+c)^(1/2)/(b*cos
(d*x+c))^(1/2)
```

Maxima [A] time = 1.94109, size = 92, normalized size = 0.86

$$\frac{3\sin(5\,dx+5\,c)+25\sin\left(\frac{3}{5}\arctan(\sin(5\,dx+5\,c),\cos(5\,dx+5\,c))\right)+150\sin\left(\frac{1}{5}\arctan(\sin(5\,dx+5\,c),\cos(5\,dx\right.}{240\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*sin(5*d*x + 5*c) + 25*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c))) + 150*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))/(sqrt(
b)*d)
```

Fricas [A] time = 2.01629, size = 144, normalized size = 1.35

$$\frac{\left(3\cos(dx+c)^4+4\cos(dx+c)^2+8\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{15\,bd\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(d*x + c)^4 + 4*cos(d*x + c)^2 + 8)*sqrt(b*cos(d*x + c))*sin(d*x
 + c)/(b*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{11}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)/sqrt(b*cos(d*x + c)), x)
```

3.172 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$\frac{3x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d\sqrt{b\cos(c+dx)}}$$

```
[Out] (3*x*Sqrt[Cos[c + d*x]])/(8*Sqrt[b*Cos[c + d*x]]) + (3*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(8*d*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(7/2)*Sin[c + d*x])
/(4*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.027159, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{3x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (3*x*Sqrt[Cos[c + d*x]])/(8*Sqrt[b*Cos[c + d*x]]) + (3*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(8*d*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(7/2)*Sin[c + d*x])
/(4*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{b\cos(c+dx)}}$$
$$= \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{b\cos(c+dx)}}$$
$$= \frac{3x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}} + \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0738192, size = 55, normalized size = 0.56

$$\frac{(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))\sqrt{\cos(c+dx)}}{32d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
/(32*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.257, size = 62, normalized size = 0.6

$$\frac{2\,(\cos(dx+c))^3\sin(dx+c)+3\,\cos(dx+c)\sin(dx+c)+3\,dx+3\,c}{8\,d}\sqrt{\cos(dx+c)}\frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(2*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)+3*d*x+3*c)*cos(d*x
+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.82529, size = 66, normalized size = 0.67

$$\frac{12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)}{32\,\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))/(sqrt(b)*d)
```

Fricas [A] time = 2.2906, size = 506, normalized size = 5.16

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\left(2\,\cos\left(dx+c\right)^{2}+3\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\,\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cdots}\right.}{16\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin
(d*x + c) - 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt
(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/8*(sqrt(b*cos(d*x + c))
*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*arctan(
sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/sqrt(b*cos(d*x + c)), x)
```

3.173 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) - (Sqrt[Cos[c +
d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0168504, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) - (Sqrt[Cos[c +
d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}\\
&= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0811437, size = 45, normalized size = 0.64

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(\cos(2(c+dx))+5)}{6d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqrt[b*Cos[c
+ d*x]])
```

Maple [A] time = 0.257, size = 40, normalized size = 0.6

$$\frac{\left(2+\left(\cos\left(dx+c\right)\right)^{2}\right)\sin\left(dx+c\right)}{3\,d}\sqrt{\cos\left(dx+c\right)}\frac{1}{\sqrt{b\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*(2+cos(d*x+c)^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.77449, size = 57, normalized size = 0.81

$$\frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)}{12\,\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(sqrt(b)*d)
```

Fricas [A] time = 1.94882, size = 115, normalized size = 1.64

$$\frac{\sqrt{b\cos\left(dx+c\right)}\left(\cos\left(dx+c\right)^{2}+2\right)\sin\left(dx+c\right)}{3\,bd\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(b*d*sqrt(cos(d*
x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/sqrt(b*cos(d*x + c)), x)
```

3.174 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=63

$$\frac{x\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0146331, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int 1\,dx}{2\sqrt{b\cos(c+dx)}}\\ &= \frac{x\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0542813, size = 45, normalized size = 0.71

$$\frac{(2(c+dx)+\sin(2(c+dx)))\sqrt{\cos(c+dx)}}{4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[b*Cos[c + d
*x]])
```

Maple [A] time = 0.244, size = 42, normalized size = 0.7

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d}\sqrt{\cos(dx+c)}\frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(cos(d*x+c)*sin(d*x+c)+d*x+c)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.76579, size = 34, normalized size = 0.54

$$\frac{2\,dx+2\,c+\sin(2\,dx+2\,c)}{4\,\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(sqrt(b)*d)
```

Fricas [A] time = 2.24993, size = 437, normalized size = 6.94

$$\left[\frac{2\,\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)-\sqrt{-b}\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx\right.}{4\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(-b)*log
(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*si
n(d*x + c) - b))/(b*d), 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x
+ c)^(3/2))))/(b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x)
```

3.175 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0068127, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0310402, size = 32, normalized size = 1.

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.246, size = 29, normalized size = 0.9

$$\frac{\sin\left(dx+c\right)\sqrt{\cos\left(dx+c\right)}}{d}\frac{1}{\sqrt{b\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.78478, size = 18, normalized size = 0.56

$$\frac{\sin\left(dx+c\right)}{\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(sqrt(b)*d)
```

Fricas [A] time = 1.82363, size = 81, normalized size = 2.53

$$\frac{\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{bd\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x)
```

3.176 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

```
[Out] (x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]]
```

Rubi [A] time = 0.0025845, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int 1\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0111623, size = 24, normalized size = 1.

$$\frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]]
```

Maple [A] time = 0.168, size = 28, normalized size = 1.2

$$\frac{dx+c}{d}\sqrt{\cos(dx+c)}\frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(d*x+c)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.56821, size = 35, normalized size = 1.46

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(sqrt(b)*d)
```

Fricas [A] time = 2.25144, size = 274, normalized size = 11.42

$$\left[-\frac{\sqrt{-b}\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)}{2\,bd},\ \frac{\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)}{\sqrt{b}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqr
t(cos(d*x + c))*sin(d*x + c) - b)/(b*d), arctan(sqrt(b*cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/(sqrt(b)*d)]
```

Sympy [A] time = 3.72132, size = 5, normalized size = 0.21

$$\frac{x}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] x/sqrt(b)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x)
```

3.177 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0072366, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {18, 3770}

$$\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0120129, size = 33, normalized size = 1.

$$\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.229, size = 42, normalized size = 1.3

$$-2\,\frac{\sqrt{\cos\left(dx+c\right)}}{d\sqrt{b\cos\left(dx+c\right)}}\operatorname{Artanh}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1
/2)
```

Maxima [B] time = 1.81523, size = 88, normalized size = 2.67

$$\frac{\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)}{2\,\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(sqrt(b)*d)
```

Fricas [A] time = 2.1238, size = 319, normalized size = 9.67

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)}{2\,\sqrt{b}d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/(sqrt(b)*d), -sqrt(-b
)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))
/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\cos\left(c+dx\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))), x)
```

3.178 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0122905, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 3767, 8}

$$\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}\\ &= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0228144, size = 32, normalized size = 1.

$$\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.238, size = 29, normalized size = 0.9

$$\frac{\sin(dx+c)}{d}\frac{1}{\sqrt{\cos(dx+c)}}\frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [B] time = 1.7695, size = 80, normalized size = 2.5

$$\frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(b\cos(2\,dx+2\,c)^2+b\sin(2\,dx+2\,c)^2+2\,b\cos(2\,dx+2\,c)+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 +
2*b*cos(2*d*x + 2*c) + b)*d)
```

Fricas [A] time = 1.81807, size = 81, normalized size = 2.53

$$\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{bd\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(3/2)), x)
```

3.179 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + Sin
[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0217717, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 3768, 3770}

$$\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + Sin
[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{2\sqrt{b\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0361573, size = 52, normalized size = 0.72

$$\frac{\sin(c+dx)+\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/
2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.25, size = 104, normalized size = 1.4

$$-\frac{1}{2d}\left((\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-(\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))/cos(d*x+c)^(3/2)/(
b*cos(d*x+c))^(1/2)
```

Maxima [B] time = 1.8258, size = 892, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
 + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) +
1)*sqrt(b)*d)
```

Fricas [A] time = 2.12803, size = 564, normalized size = 7.83

$$\left[\frac{\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(d}{4bd\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x +
 c)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(5/2)), x)
```

3.180 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + Sin[c + d*x]^3/(
3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0179936, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {18, 3767}

$$\frac{\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + Sin[c + d*x]^3/(
3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\mathrm{Subst}\left(\int\left(1+x^2\right)dx,x,-\tan(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}\\ &= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0759968, size = 45, normalized size = 0.64

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sqrt[b*Cos[c + d*
x]])
```

Maple [A] time = 0.251, size = 42, normalized size = 0.6

$$\frac{\sin(dx+c)\left(2(\cos(dx+c))^2+1\right)}{3d}(\cos(dx+c))^{-\frac{5}{2}}\frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*sin(d*x+c)*(2*cos(d*x+c)^2+1)/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 1.81671, size = 397, normalized size = 5.67

$$\frac{4((3\cos(2dx+2c)\ \cdots}{3\left(2(3\cos(4dx+4c)+3\cos(2dx+2c)+1)\cos(6dx+6c)+\cos(6dx+6c)^2+6(3\cos(2dx+2c)+1)\cos(4d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6
*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + s
in(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)
^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*
d*x + 2*c) + 1)*sqrt(b)*d)
```

Fricas [A] time = 1.90385, size = 117, normalized size = 1.67

$$\frac{\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+1\right)\sin(dx+c)}{3bd\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(b*d*cos(d*x +
 c)^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)), x)
```

3.181 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=107

$$\frac{3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8d\sqrt{b\cos(c+dx)}}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c + d*x]]) + S
in[c + d*x]/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (3*Sin[c + d*x]
)/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0349813, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 3768, 3770}

$$\frac{3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c + d*x]]) + S
in[c + d*x]/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (3*Sin[c + d*x]
)/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^5(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{b\cos(c+dx)}}$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)}{8\sqrt{b\,\mathrm{co}}}$$

$$= \frac{3\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{3\,\mathrm{s}}{8d\cos^{\frac{3}{2}}(c+}$$

Mathematica [A] time = 0.0785357, size = 66, normalized size = 0.62

$$\frac{\sin(c+dx)\left(3\cos^2(c+dx)+2\right)+3\cos^4(c+dx)\tanh^{-1}(\sin(c+dx))}{8d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[c + d*x]^2)*Sin[c + d*
x])/(8*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.282, size = 121, normalized size = 1.1

$$\frac{1}{8\,d}\left(3\,(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-3\,(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+
c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2
*sin(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [B] time = 1.89037, size = 2236, normalized size = 20.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*s
in(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*
(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x +
 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
 + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
```

```
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^
2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*c
os(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4
*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6
*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*s
in(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(
2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1
)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d
*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2
 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d
*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c
) + 1)*sqrt(b)*d)
```

Fricas [A] time = 2.23464, size = 633, normalized size = 5.92

$$\left[\frac{3\sqrt{b}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^{2}+\right.}{16\,bd\cos\left(dx+c\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*si
n(d*x + c))/(b*d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(d*x +
c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - sqrt(b*c
os(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*c
```

```
os(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(9/2)), x)
```

3.182 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4bd\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (3*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + (3*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(7/2)*Sin[c + d
*x])/(4*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0290272, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{3x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4bd\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(3/2),x]
```



```
[Out] (3*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + (3*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(7/2)*Sin[c + d
*x])/(4*b*d*Sqrt[b*Cos[c + d*x]])
```


Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4b\sqrt{b\cos(c+dx)}}$$
$$= \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8b\sqrt{b\cos(c+dx)}}$$
$$= \frac{3x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0837066, size = 55, normalized size = 0.51

$$\frac{(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))\cos^{\frac{3}{2}}(c+dx)}{32d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
/(32*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.178, size = 62, normalized size = 0.6

$$\frac{2\,(\cos(dx+c))^3\sin(dx+c)+3\,\cos(dx+c)\sin(dx+c)+3\,dx+3\,c}{8\,d}(\cos(dx+c))^{\frac{3}{2}}(b\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/8/d*cos(d*x+c)^(3/2)*(2*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)+3
*d*x+3*c)/(b*cos(d*x+c))^(3/2)
```

Maxima [A] time = 1.86491, size = 66, normalized size = 0.62

$$\frac{12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)}{32\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))/(b^(3/2)*d)
```

Fricas [A] time = 2.32015, size = 512, normalized size = 4.79

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\left(2\,\cos\left(dx+c\right)^2+3\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\,\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-}\right.}{16\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin
(d*x + c) - 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt
(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/8*(sqrt(b*cos(d*x + c
))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*arcta
n(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{11}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)/(b*cos(d*x + c))^(3/2), x)
```

3.183 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) - (Sqrt[Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0190838, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) - (Sqrt[Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}\\
&= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0668974, size = 45, normalized size = 0.59

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(\cos(2(c+dx))+5)}{6d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*(b*Cos[c + d*
x])^(3/2))
```

Maple [A] time = 0.157, size = 40, normalized size = 0.5

$$\frac{\left(2+\left(\cos\left(dx+c\right)\right)^{2}\right)\sin\left(dx+c\right)}{3\,d}\left(\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\cos\left(dx+c\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/3/d*(2+cos(d*x+c)^2)*sin(d*x+c)*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [A] time = 1.81652, size = 57, normalized size = 0.75

$$\frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)}{12\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(b^(3/2)*d)
```

Fricas [A] time = 1.89902, size = 117, normalized size = 1.54

$$\frac{\sqrt{b\cos\left(dx+c\right)}\left(\cos\left(dx+c\right)^{2}+2\right)\sin\left(dx+c\right)}{3\,b^{2}d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(b^2*d*sqrt(cos(
d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(3/2), x)
```

3.184 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0164085, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int 1\,dx}{2b\sqrt{b\cos(c+dx)}}\\ &= \frac{x\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0391866, size = 45, normalized size = 0.65

$$\frac{(2(c+dx)+\sin(2(c+dx)))\cos^{\frac{3}{2}}(c+dx)}{4d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*(b*Cos[c + d*x])
^(3/2))
```

Maple [A] time = 0.161, size = 42, normalized size = 0.6

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d}(\cos(dx+c))^{\frac{3}{2}}(b\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*cos(d*x+c)^(3/2)*(cos(d*x+c)*sin(d*x+c)+d*x+c)/(b*cos(d*x+c))^(3/2)
```

Maxima [A] time = 1.78568, size = 34, normalized size = 0.49

$$\frac{2dx+2c+\sin(2dx+2c)}{4b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(3/2)*d)
```

Fricas [A] time = 2.26512, size = 443, normalized size = 6.42

$$\left[\frac{2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)-\sqrt{-b}\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c\right.}{4b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(-b)*log
(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*si
n(d*x + c) - b))/(b^2*d), 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2))))/(b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2), x)
```

3.185 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0072789, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0405547, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.158, size = 29, normalized size = 0.8

$$\frac{\sin(dx+c)}{d}(\cos(dx+c))^{\frac{3}{2}}(b\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(3/2)*sin(d*x+c)/(b*cos(d*x+c))^(3/2)
```

Maxima [A] time = 1.74594, size = 18, normalized size = 0.51

$$\frac{\sin(dx+c)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(b^(3/2)*d)
```

Fricas [A] time = 1.83186, size = 84, normalized size = 2.4

$$\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{b^2d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2), x)
```

3.186 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

```
[Out] (x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.002648, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int 1\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{x\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0160469, size = 24, normalized size = 0.89

$$\frac{x\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (x*Cos[c + d*x]^(3/2))/(b*Cos[c + d*x])^(3/2)
```

Maple [A] time = 0.145, size = 28, normalized size = 1.

$$\frac{dx+c}{d}\left(\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\cos\left(dx+c\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(d*x+c)*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [A] time = 1.55606, size = 35, normalized size = 1.3

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(3/2)*d)
```

Fricas [A] time = 2.24647, size = 277, normalized size = 10.26

$$\left[-\frac{\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)}{2\,b^{2}d},\ \frac{\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)}{b^{\frac{3}{2}}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqr
t(cos(d*x + c))*sin(d*x + c) - b)/(b^2*d), arctan(sqrt(b*cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/(b^(3/2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^{\frac{3}{2}}}{(b\cos (dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2), x)
```

3.187 $\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0084481, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0131611, size = 33, normalized size = 0.92

$$\frac{\cos^{\frac{3}{2}}(c+dx)\tanh^{-1}(\sin(c+dx))}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^(3/2))/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.23, size = 42, normalized size = 1.2

$$-2\frac{\left(\cos\left(dx+c\right)\right)^{3/2}}{d\left(b\cos\left(dx+c\right)\right)^{3/2}}\operatorname{Artanh}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3
/2)
```

Maxima [B] time = 1.78566, size = 88, normalized size = 2.44

$$\frac{\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\sin\left(dx+c\right)+1\right)}{2b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(3/2)*d)
```

Fricas [A] time = 2.19227, size = 321, normalized size = 8.92

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)}{2b^{\frac{3}{2}}d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/(b^(3/2)*d), -sqrt(-b
)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))
/(b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2), x)
```

3.188 $\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] Sin[c + d*x]/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.01258, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 3767, 8}

$$\frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] Sin[c + d*x]/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}\\ &= \frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0150371, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.233, size = 29, normalized size = 0.8

$$\frac{\sin(dx+c)}{d}\sqrt{\cos(dx+c)}\,(b\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(1/2)*sin(d*x+c)/(b*cos(d*x+c))^(3/2)
```

Maxima [B] time = 1.76311, size = 90, normalized size = 2.57

$$\frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)^
2 + 2*b^2*cos(2*d*x + 2*c) + b^2)*d)
```

Fricas [A] time = 1.85864, size = 84, normalized size = 2.4

$$\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{b^2d\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(3/2)*sqrt(cos(d*x + c))), x)
```

3.189 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + S
in[c + d*x]/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.021936, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 3768, 3770}

$$\frac{\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + S
in[c + d*x]/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{2b\sqrt{b\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0374828, size = 52, normalized size = 0.67

$$\frac{\sin(c+dx)+\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x
]]*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.167, size = 104, normalized size = 1.3

$$-\frac{1}{2d}\left((\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-(\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))/cos(d*x+c)^(1/2)/(
b*cos(d*x+c))^(3/2)
```

Maxima [B] time = 1.85117, size = 905, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)
^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin
(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2
*d*x + 2*c) + b)*sqrt(b)*d)
```

Fricas [A] time = 2.29512, size = 570, normalized size = 7.31

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4\,b^2d\cos\left(dx+c\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x
 + c)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.190 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] Sin[c + d*x]/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + Sin[c + d*x]^3
/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.01873, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {18, 3767}

$$\frac{\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] Sin[c + d*x]/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + Sin[c + d*x]^3
/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= -\frac{\sqrt{\cos(c+dx)}\,\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}\\
&= \frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0256597, size = 45, normalized size = 0.59

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*(b*Cos[c + d*x])^
(3/2))
```

Maple [A] time = 0.158, size = 42, normalized size = 0.6

$$\frac{\sin(dx+c)\left(2\left(\cos(dx+c)\right)^2+1\right)}{3d}\left(\cos(dx+c)\right)^{-\frac{3}{2}}\left(b\cos(dx+c)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/3/d*sin(d*x+c)*(2*cos(d*x+c)^2+1)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2)
```

Maxima [B] time = 1.80497, size = 420, normalized size = 5.53

$$\frac{4\left((3\cos(2dx+\right.}{3\left(b\cos(6dx+6c)^2+9b\cos(4dx+4c)^2+9b\cos(2dx+2c)^2+b\sin(6dx+6c)^2+9b\sin(4dx+4c)^2+18b\sin(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*c
os(2*d*x + 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x
 + 4*c) + 3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2
*c) + b)*cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) +
b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)*d)
```

Fricas [A] time = 1.9584, size = 120, normalized size = 1.58

$$\frac{\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+1\right)\sin(dx+c)}{3b^2d\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(b^2*d*cos(d*x
 + c)^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.191 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{3\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b*d*Sqrt[b*Cos[c + d*x]]) +
 Sin[c + d*x]/(4*b*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (3*Sin[c +
d*x])/(8*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0366223, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 3768, 3770}

$$\frac{3\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b*d*Sqrt[b*Cos[c + d*x]]) +
 Sin[c + d*x]/(4*b*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (3*Sin[c +
d*x])/(8*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^5(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4b\sqrt{b\cos(c+dx)}}$$

$$= \frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{3\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos\right.}{}}{}$$

$$= \frac{3\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{}{8bd}$$

Mathematica [A] time = 0.0551507, size = 66, normalized size = 0.57

$$\frac{\sin(c+dx)\left(3\cos^2(c+dx)+2\right)+3\cos^4(c+dx)\tanh^{-1}(\sin(c+dx))}{8d\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[c + d*x]^2)*Sin[c + d*
x])/(8*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.191, size = 121, normalized size = 1.

$$\frac{1}{8d}\left(3(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-3(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+
c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2
*sin(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [B] time = 1.9137, size = 2267, normalized size = 19.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*s
in(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*
(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x +
 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
 + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
```

```
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^
2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*c
os(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4
*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6
*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*s
in(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(
2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))/((b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4
*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6
*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) +
4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*
cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(
4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d
*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*
c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)*d)
```

Fricas [A] time = 2.38972, size = 639, normalized size = 5.51

$$\left[\frac{3\,\sqrt{b}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\,\sqrt{b\cos\left(dx+c\right)}\left(3\,\cos\left(dx+c\right)^{2}+2\right)}{16\,b^{2}d\cos\left(dx+c\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*si
n(d*x + c))/(b^2*d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(d*x
+ c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - sqrt(b
*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2
```

```
*d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(7/2)), x)
```

3.192 $\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (3*x*Sqrt[Cos[c + d*x]])/(8*b^2*Sqrt[b*Cos[c + d*x]]) + (3*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(7/2)*Sin[c
 + d*x])/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0278996, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{3x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(13/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (3*x*Sqrt[Cos[c + d*x]])/(8*b^2*Sqrt[b*Cos[c + d*x]]) + (3*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(7/2)*Sin[c
 + d*x])/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{3x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0884706, size = 58, normalized size = 0.54

$$\frac{(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))\sqrt{\cos(c+dx)}}{32b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(13/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
/(32*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.178, size = 62, normalized size = 0.6

$$\frac{2\left(\cos(dx+c)\right)^3\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3\,dx+3\,c}{8\,d}\left(\cos(dx+c)\right)^{\frac{5}{2}}\left(b\cos(dx+c)\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/8/d*cos(d*x+c)^(5/2)*(2*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)+3
*d*x+3*c)/(b*cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.81202, size = 66, normalized size = 0.62

$$\frac{12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)}{32\,b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))/(b^(5/2)*d)
```

Fricas [A] time = 2.66179, size = 512, normalized size = 4.79

$$\left[\frac{2\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+3\right)\sqrt{\cos(dx+c)}\sin(dx+c)-3\sqrt{-b}\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\ldots}\right.}{16b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin
(d*x + c) - 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt
(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/8*(sqrt(b*cos(d*x + c
))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*arcta
n(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(13/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{13}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(13/2)/(b*cos(d*x + c))^(5/2), x)
```

3.193 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) - (Sqrt[Cos[
c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0176209, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) - (Sqrt[Cos[
c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}\\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0649478, size = 48, normalized size = 0.63

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(\cos(2(c+dx))+5)}{6b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*b^2*d*Sqrt[b*Co
s[c + d*x]])
```

Maple [A] time = 0.155, size = 40, normalized size = 0.5

$$\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3d}(\cos(dx+c))^{\frac{5}{2}}(b\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*(2+cos(d*x+c)^2)*sin(d*x+c)*cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.82482, size = 57, normalized size = 0.75

$$\frac{\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)}{12b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(b^(5/2)*d)
```

Fricas [A] time = 2.20843, size = 117, normalized size = 1.54

$$\frac{\sqrt{b\cos(dx+c)}\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3b^3d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(b^3*d*sqrt(cos(
d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{11}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)/(b*cos(d*x + c))^(5/2), x)
```

3.194 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*S
in[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0148098, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*S
in[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int 1\,dx}{2b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{x\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0443976, size = 48, normalized size = 0.7

$$\frac{(2(c+dx)+\sin(2(c+dx)))\sqrt{\cos(c+dx)}}{4b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*b^2*d*Sqrt[b*Cos[c
 + d*x]])
```

Maple [A] time = 0.158, size = 42, normalized size = 0.6

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d}(\cos(dx+c))^{\frac{5}{2}}(b\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*cos(d*x+c)^(5/2)*(cos(d*x+c)*sin(d*x+c)+d*x+c)/(b*cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.77892, size = 34, normalized size = 0.49

$$\frac{2dx+2c+\sin(2dx+2c)}{4b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(5/2)*d)
```

Fricas [A] time = 2.39396, size = 443, normalized size = 6.42

$$\left[\frac{2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)-\sqrt{-b}\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx\right.}{4b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(-b)*log
(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*si
n(d*x + c) - b))/(b^3*d), 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2))))/(b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2), x)
```

3.195 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0072171, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0258133, size = 35, normalized size = 1.

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.152, size = 29, normalized size = 0.8

$$\frac{\sin\left(dx+c\right)}{d}\left(\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(b\cos\left(dx+c\right)\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/d*sin(d*x+c)*cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.79222, size = 18, normalized size = 0.51

$$\frac{\sin\left(dx+c\right)}{b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(b^(5/2)*d)
```

Fricas [A] time = 1.68191, size = 84, normalized size = 2.4

$$\frac{\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b^{3}d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2), x)
```

3.196 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

```
[Out] (x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0026813, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int 1\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{x\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0124874, size = 24, normalized size = 0.89

$$\frac{x\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (x*Cos[c + d*x]^(5/2))/(b*Cos[c + d*x])^(5/2)
```

Maple [A] time = 0.136, size = 28, normalized size = 1.

$$\frac{dx+c}{d}(\cos(dx+c))^{\frac{5}{2}}(b\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/d*(d*x+c)*cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.57901, size = 35, normalized size = 1.3

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(5/2)*d)
```

Fricas [A] time = 1.85471, size = 277, normalized size = 10.26

$$\left[-\frac{\sqrt{-b}\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)}{2\,b^3d},\ \frac{\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)}{b^{\frac{5}{2}}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqr
t(cos(d*x + c))*sin(d*x + c) - b)/(b^3*d), arctan(sqrt(b*cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/(b^(5/2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2), x)
```

3.197 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0077614, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0246913, size = 33, normalized size = 0.92

$$\frac{\cos^{\frac{5}{2}}(c+dx)\tanh^{-1}(\sin(c+dx))}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^(5/2))/(d*(b*Cos[c + d*x])^(5/2))
```

Maple [A] time = 0.157, size = 42, normalized size = 1.2

$$-2\frac{(\cos(dx+c))^{5/2}}{d(b\cos(dx+c))^{5/2}}\operatorname{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/d*cos(d*x+c)^(5/2)*arctanh((-1+cos(d*x+c))/sin(d*x+c))/(b*cos(d*x+c))^(5
/2)
```

Maxima [B] time = 1.79114, size = 88, normalized size = 2.44

$$\frac{\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)}{2b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(5/2)*d)
```

Fricas [A] time = 1.8501, size = 321, normalized size = 8.92

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2b^{\frac{5}{2}}d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/(b^(5/2)*d), -sqrt(-b
)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))
/(b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(5/2), x)
```

3.198 $\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] Sin[c + d*x]/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0132257, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3767, 8}

$$\frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] Sin[c + d*x]/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\
&= -\frac{\sqrt{\cos(c+dx)}\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}\\
&= \frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0170582, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(5/2))
```

Maple [A] time = 0.237, size = 29, normalized size = 0.8

$$\frac{\sin(dx+c)}{d}(\cos(dx+c))^{\frac{3}{2}}(b\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(3/2)*sin(d*x+c)/(b*cos(d*x+c))^(5/2)
```

Maxima [B] time = 1.74199, size = 90, normalized size = 2.57

$$\frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(b^3\cos(2\,dx+2\,c)^2+b^3\sin(2\,dx+2\,c)^2+2\,b^3\cos(2\,dx+2\,c)+b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)^
2 + 2*b^3*cos(2*d*x + 2*c) + b^3)*d)
```

Fricas [A] time = 1.66569, size = 84, normalized size = 2.4

$$\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{b^3d\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c))^(5/2), x)
```

3.199 $\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) +
 Sin[c + d*x]/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0220886, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 3768, 3770}

$$\frac{\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) +
 Sin[c + d*x]/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{2b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0459511, size = 55, normalized size = 0.71

$$\frac{\sqrt{b\cos(c+dx)}\left(\sin(c+dx)+\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2b^3d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x])
)/(2*b^3*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.245, size = 102, normalized size = 1.3

$$\frac{1}{2\,d}\left((\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-(\cos(dx+c))^2\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^2
*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+sin(d*x+c))*cos(d*x+c)^(1/2)/(b
*cos(d*x+c))^(5/2)
```

Maxima [B] time = 1.86105, size = 929, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x +
2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d
*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)*d)
```

Fricas [A] time = 1.91897, size = 570, normalized size = 7.31

$$\left[\frac{\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(d}{4b^3d\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x
 + c)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(5/2)*sqrt(cos(d*x + c))), x)
```

3.200 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] Sin[c + d*x]/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + Sin[c + d*x]
^3/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.01834, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {18, 3767}

$$\frac{\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] Sin[c + d*x]/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + Sin[c + d*x]
^3/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}\\ &= \frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0282328, size = 45, normalized size = 0.59

$$\frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*(b*Cos[c + d*x])^
(5/2))
```

Maple [A] time = 0.161, size = 42, normalized size = 0.6

$$\frac{\sin(dx+c)\left(2\,(\cos(dx+c))^2+1\right)}{3\,d}\frac{1}{\sqrt{\cos(dx+c)}}(b\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*sin(d*x+c)*(2*cos(d*x+c)^2+1)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2)
```

Maxima [B] time = 1.87608, size = 463, normalized size = 6.09

$$\frac{4\,((3}{3\left(b^2\cos(6\,dx+6\,c)^2+9\,b^2\cos(4\,dx+4\,c)^2+9\,b^2\cos(2\,dx+2\,c)^2+b^2\sin(6\,dx+6\,c)^2+9\,b^2\sin(4\,dx+4\,c)^2+18\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))/((b^2*cos(6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9
*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2
 + 18*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*
b^2*cos(2*d*x + 2*c) + b^2 + 2*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x +
2*c) + b^2)*cos(6*d*x + 6*c) + 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x +
 4*c) + 6*(b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*s
qrt(b)*d)
```

Fricas [A] time = 1.66222, size = 120, normalized size = 1.58

$$\frac{\sqrt{b\cos(dx+c)}\left(2\,\cos(dx+c)^2+1\right)\sin(dx+c)}{3\,b^3d\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(b^3*d*cos(d*x
 + c)^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.201 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{3\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b^2*d*Sqrt[b*Cos[c + d*x]])
 + Sin[c + d*x]/(4*b^2*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (3*Sin[
c + d*x])/(8*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0365752, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 3768, 3770}

$$\frac{3\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b^2*d*Sqrt[b*Cos[c + d*x]])
 + Sin[c + d*x]/(4*b^2*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (3*Sin[
c + d*x])/(8*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^5(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{3\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c\right.}}{8}$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{}{8b^2d\,c}$$

Mathematica [A] time = 0.06647, size = 66, normalized size = 0.57

$$\frac{\sin(c+dx)\left(3\cos^2(c+dx)+2\right)+3\cos^4(c+dx)\tanh^{-1}(\sin(c+dx))}{8d\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[c + d*x]^2)*Sin[c + d*
x])/(8*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
```

Maple [A] time = 0.191, size = 121, normalized size = 1.

$$-\frac{1}{8d}\left(3\,(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-3\,(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-3*cos(d*x
+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+c)^2*sin(d*x+c)-
2*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [B] time = 1.9245, size = 2334, normalized size = 20.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*s
in(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*
(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x +
 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
 + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
```

```
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^
2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*c
os(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4
*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6
*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*s
in(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(
2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))/((b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*d
*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*s
in(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 +
2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c)
 + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*
c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x +
4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
 + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c))*sqrt(b)*d)
```

Fricas [A] time = 1.98616, size = 639, normalized size = 5.51

$$\left[\frac{3\sqrt{b}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^{2}+\right.}{16b^{3}d\cos\left(dx+c\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*si
n(d*x + c))/(b^3*d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(d*x
+ c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - sqrt(b
```

```
*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3
*d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.202 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (4 +
 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rubi [A] time = 0.0261172, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (4 +
 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}}$$

$$= -\frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{1}{6}(10+3m);\cos^2(c+dx)\right)\text{si}}{d(4+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.108037, size = 82, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(m+\frac{4}{3}\right);\frac{1}{2}\left(m+\frac{10}{3}\right);\cos^2(c+dx)\right)}{d\left(m+\frac{4}{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*Hypergeometric2
F1[1/2, (4/3 + m)/2, (10/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d
*(4/3 + m)))
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m \sqrt[3]{b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{1}{3}} \cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{1}{3}} \cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\cos(c+dx)}\cos^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*cos(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

3.203 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(10*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.019846, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(10*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx &= \frac{\int (b\cos(c+dx))^{7/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0622966, size = 63, normalized size = 1.09

$$-\frac{3\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/
2, 5/3, 8/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(10*d)
```

Maple [F] time = 0.213, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^2 \sqrt[3]{b\cos (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (dx+c))^{\frac{1}{3}} \cos (dx+c)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos (dx+c))^{\frac{1}{3}} \cos (dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

3.204 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(7*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0204174, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(7*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx &= \frac{\int (b\cos(c+dx))^{4/3}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0534614, size = 58, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(7*b*d)
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int \cos(dx+c)\sqrt[3]{b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

3.205 $\int \sqrt[3]{b\cos(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.013692, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}\,dx = -\frac{3(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0364869, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Co
s[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F] time = 0.232, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3), x)
```

3.206 $\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx$

Optimal. Leaf size=53

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0244257, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x],x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx &= b\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx\\ &= -\frac{3\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0484639, size = 54, normalized size = 1.02

$$-\frac{3b\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x],x]
```

```
[Out] (-3*b*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\cos(dx+c)}\sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*sec(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*sec(d*x+c),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.207 $\int \sqrt[3]{b\cos(c+dx)}\sec^2(c+dx)\,dx$

Optimal. Leaf size=56

$$\frac{3b\sin(c+dx)\,_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

```
[Out] (3*b*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0308035, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b\sin(c+dx)\,_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\cos(c+dx)}\sec^2(c+dx)\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx\\ &= \frac{3b\,_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0588324, size = 56, normalized size = 1.

$$\frac{3b\sqrt{\sin^2(c+dx)}\csc(c+dx)\,_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*Csc[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.147, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\cos(dx+c)}\,(\sec(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.208 $\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}$$

```
[Out] (3*b^2*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d
*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0316683, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^3,x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d
*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{8/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0482712, size = 63, normalized size = 1.09

$$\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\sec^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^3,x]
```

```
[Out] (3*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Co
s[c + d*x]^2]*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(5*d)
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \sqrt[3]{b \cos(dx+c)}\,(\sec(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \cos(dx+c))^{\frac{1}{3}} \sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b \cos(dx+c))^{\frac{1}{3}} \sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (dx+c))^{\frac{1}{3}} \sec (dx+c)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

3.209 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (5 +
 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rubi [A] time = 0.0278641, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (5 +
 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx = \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{2}{3}+m}(c+dx)\,dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(11+3m);\cos^2(c+dx)\right)\mathrm{s}}{d(5+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.117181, size = 82, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(m+\frac{5}{3}\right);\frac{1}{2}\left(m+\frac{11}{3}\right);\cos^2(c+dx)\right)}{d\left(m+\frac{5}{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*Hypergeometric2
F1[1/2, (5/3 + m)/2, (11/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d
*(5/3 + m)))
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (b\cos(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

3.210 $\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{11/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)}{11b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(11*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0199932, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{11/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)}{11b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(11*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx &= \frac{\int (b\cos(c+dx))^{8/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{11/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)\sin(c+dx)}{11b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.061232, size = 63, normalized size = 1.09

$$-\frac{3\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)}{11d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/
2, 11/6, 17/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(11*d)
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int (\cos (dx + c))^2 (b \cos (dx + c))^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \cos (dx + c))^{\frac{2}{3}} \cos (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \cos (dx + c))^{\frac{2}{3}} \cos (dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

3.211 $\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0199726, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx &= \frac{\int (b\cos(c+dx))^{5/3}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0543538, size = 58, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Co
s[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(8*b*d)
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(b\cos(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}} \cos(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

3.212 $\int (b\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0144588, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3}\,dx=-\frac{3(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0373891, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(5*d)
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3), x)
```

3.213 $\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx$

Optimal. Leaf size=55

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0265928, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x],x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx &= b\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx\\ &= -\frac{3(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.045929, size = 56, normalized size = 1.02

$$-\frac{3b\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x],x]
```

```
[Out] (-3*b*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}} \sec(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*sec(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}} \sec(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{2}{3}} \sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

3.214 $\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx$

Optimal. Leaf size=54

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[Out] (3*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*
Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0360375, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*
Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0589294, size = 54, normalized size = 1.

$$\frac{3b\sqrt{\sin^2(c+dx)}\csc(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(d*(b*Cos[c + d*x])^(1/3))
```

Maple [F] time = 0.143, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}} (\sec(dx+c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}} \sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c))^{\frac{2}{3}} \sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

3.215 $\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

```
[Out] (3*b^2*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d
*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0356666, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^3,x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d
*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0430168, size = 63, normalized size = 1.09

$$\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\sec^2(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^3,x]
```

```
[Out] (3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Co
s[c + d*x]^2]*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int (b\cos (dx+c))^{\frac{2}{3}} (\sec (dx+c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

3.216 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx$

Optimal. Leaf size=83

$$-\frac{3b\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*b*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7
 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[Si
n[c + d*x]^2])
```

Rubi [A] time = 0.0263632, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$-\frac{3b\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*b*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7
 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[Si
n[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx &= \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}}\\ &= -\frac{3b\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+3m);\cos^2(c+dx)\right)}{d(7+3m)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.177545, size = 82, normalized size = 0.99

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^{4/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(m+\frac{7}{3}\right);\frac{1}{2}\left(m+\frac{13}{3}\right);\cos^2(c+dx)\right)}{d\left(m+\frac{7}{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*Hypergeometric2
F1[1/2, (7/3 + m)/2, (13/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d
*(7/3 + m)))
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (b\cos(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{1}{3}}\,b\cos(dx+c)^m\cos(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)^m*cos(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

3.217 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(13*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0198179, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3),x]
```


```
[Out] (-3*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(13*b^3*d*Sqrt[Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx &= \frac{\int (b\cos(c+dx))^{10/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)\sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.079578, size = 63, normalized size = 1.09

$$-\frac{3\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*Hypergeometric2F1[1/
2, 13/6, 19/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(13*d)
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^{2} (b\cos (dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (dx+c))^{\frac{4}{3}} \cos (dx+c)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos (dx+c))^{\frac{1}{3}} b\cos (dx+c)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

3.218 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0196422, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx &= \frac{\int (b\cos(c+dx))^{7/3}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0743023, size = 58, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Co
s[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(10*b*d)
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(b\cos(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c))^{\frac{1}{3}}\,b\cos(dx+c)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}} \cos(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

3.219 $\int (b\cos(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0143119, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}\,dx = -\frac{3(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0028167, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(7*d)
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{1}{3}}\,b\cos(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3), x)
```

3.220 $\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx$

Optimal. Leaf size=55

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0251622, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x],x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx &= b\int \sqrt[3]{b\cos(c+dx)}\,dx\\ &= -\frac{3(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0227077, size = 56, normalized size = 1.02

$$-\frac{3b\sqrt{\sin^2(c+dx)}\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x],x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3,
Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F] time = 0.157, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}} \sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*sec(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}} \sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{1}{3}} b\cos(dx+c)\sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

3.221 $\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx$

Optimal. Leaf size=54

$$-\frac{3b\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0355324, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$-\frac{3b\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx\\ &= -\frac{3b\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0279344, size = 56, normalized size = 1.04

$$-\frac{3b^2\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sqrt[
Sin[c + d*x]^2])/(d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.182, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}} (\sec(dx+c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}} \sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c))^{\frac{1}{3}} b\cos(dx+c)\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

3.222 $\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

```
[Out] (3*b^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d
*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0367279, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^3,x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d
*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0609135, size = 58, normalized size = 1.

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\csc(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^3,x]
```

```
[Out] (3*b^2*Csc[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sqrt[
Sin[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}} (\sec(dx+c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}} \sec(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^{\frac{1}{3}} b\cos(dx+c)\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

3.223 $\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c
 + d*x]^2])
```

Rubi [A] time = 0.0263166, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$-\frac{3\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c
 + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\cos(c+dx)}}\\ &= -\frac{3\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\cos^2(c+dx)\right)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.114762, size = 82, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(m+\frac{2}{3}\right);\frac{1}{2}\left(m+\frac{8}{3}\right);\cos^2(c+dx)\right)}{d\left(m+\frac{2}{3}\right)\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*Hypergeometric2F1[1/2, (2/3 + m)/2, (8
/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(2/3 + m)*(b*Cos[c + d*
x])^(1/3)))
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m \frac{1}{\sqrt[3]{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral(cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

3.224 $\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.019733, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= \frac{\int (b\cos(c+dx))^{5/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0755762, size = 63, normalized size = 1.09

$$-\frac{3\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*
x]^2]*Sqrt[Sin[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int (\cos (dx + c))^2 \frac{1}{\sqrt[3]{b \cos (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx + c)^2}{(b \cos (dx + c))^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \cos (dx + c))^{\frac{2}{3}} \cos (dx + c)}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

3.225 $\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0195851, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= \frac{\int (b\cos(c+dx))^{2/3}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0049375, size = 58, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(5*b*d)
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \cos(dx+c)\frac{1}{\sqrt[3]{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c))^{\frac{2}{3}}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

3.226 $\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0144522, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx=-\frac{3(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0034088, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-1/3),x]
```

```
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sqrt[Sin[
c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(1/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c))^{\frac{2}{3}}}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*cos(c + d*x))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(-1/3), x)
```

3.227 $\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Co
s[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0259716, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Co
s[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= b\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx\\ &= \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0490312, size = 54, normalized size = 1.02

$$\frac{3b\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*b*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(d*(b*Cos[c + d*x])^(4/3))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \sec \left(dx+c\right) \frac{1}{\sqrt[3]{b\cos \left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(dx+c\right)}{\left(b\cos \left(dx+c\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\cos \left(dx+c\right)\right)^{\frac{2}{3}} \sec \left(dx+c\right)}{b\cos \left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(c+dx\right)}{\sqrt[3]{b\cos \left(c+dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)/(b*cos(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

3.228 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

```
[Out] (3*b*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0338812, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*b*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0560049, size = 58, normalized size = 1.04

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sqrt[
Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(7/3))
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \left(\sec \left(dx+c\right)\right)^{2}\frac{1}{\sqrt[3]{b\cos \left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(dx+c\right)^{2}}{\left(b\cos \left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\cos \left(dx+c\right)\right)^{\frac{2}{3}}\sec \left(dx+c\right)^{2}}{b\cos \left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}\left(c+dx\right)}{\sqrt[3]{b\cos \left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*cos(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

3.229 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}$$

```
[Out] (3*b^2*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0324211, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{10/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.10872, size = 58, normalized size = 1.

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\csc(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(7/3))
```

Maple [F] time = 0.157, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{3} \frac{1}{\sqrt[3]{b\cos (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{3}}{(b\cos (dx+c))^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c)^{3}}{b\cos (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}(c+dx)}{\sqrt[3]{b\cos (c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(b*cos(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

3.230 $\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c
 + d*x]^2])
```

Rubi [A] time = 0.0246889, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$-\frac{3\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c
 + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= \frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{-\frac{2}{3}+m}(c+dx)\,dx}{(b\cos(c+dx))^{2/3}}\\ &= -\frac{3\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1+3m);\frac{1}{6}(7+3m);\cos^2(c+dx)\right)\sin(c+dx)}{d(1+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.113248, size = 82, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(m+\frac{1}{3}\right);\frac{1}{2}\left(m+\frac{7}{3}\right);\cos^2(c+dx)\right)}{d\left(m+\frac{1}{3}\right)(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*Hypergeometric2F1[1/2, (1/3 + m)/2, (7
/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1/3 + m)*(b*Cos[c + d*
x])^(2/3)))
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (b\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral(cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

3.231 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(7*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0200931, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(7*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= \frac{\int (b\cos(c+dx))^{4/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0762718, size = 63, normalized size = 1.09

$$-\frac{3\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d
*x]^2]*Sqrt[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.16, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\cos(dx+c))^{-\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.232 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(4*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0201501, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(4*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= \frac{\int \sqrt[3]{b\cos(c+dx)}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0202992, size = 58, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Co
s[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4*b*d)
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \cos \left(dx+c\right) \left(b\cos \left(dx+c\right)\right)^{-\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos \left(dx+c\right)}{\left(b\cos \left(dx+c\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\cos \left(dx+c\right)\right)^{\frac{1}{3}}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.233 $\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0151921, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx=-\frac{3\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0225721, size = 53, normalized size = 0.95

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-2/3),x]
```

```
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sqrt[Sin[
c + d*x]^2])/(d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(1/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c))^{\frac{1}{3}}}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*cos(c + d*x))**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(-2/3), x)
```

3.234 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=55

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

```
[Out] (3*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*
Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0263834, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*
Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= b\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx\\ &= \frac{3\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0479952, size = 56, normalized size = 1.02

$$\frac{3b\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d(b\cos(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b*Cot[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(5/3))
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int \sec\left(dx+c\right)\left(b\cos\left(dx+c\right)\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)}{b\cos\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral(sec(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.235 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=56

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}$$

```
[Out] (3*b*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*(
b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0373004, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*(
b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{8/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0583944, size = 58, normalized size = 1.04

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d(b\cos(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sqrt[
Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(8/3))
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (b\cos(dx+c))^{-\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*cos(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^2}{(b\cos (dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.236 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\sin(c+dx)\,_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{8/3}}$$

```
[Out] (3*b^2*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*
d*(b*Cos[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0453945, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b^2\sin(c+dx)\,_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*
d*(b*Cos[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{11/3}}\,dx\\ &= \frac{3b^2\,_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.103077, size = 58, normalized size = 1.

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\csc(c+dx)\,_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)}{8d(b\cos(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(8/3))
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{3} (b\cos (dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{3}}{(b\cos (dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos (dx+c))^{\frac{1}{3}} \sec (dx+c)^{3}}{b\cos (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}(c+dx)}{(b\cos (c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(b*cos(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^3}{(b\cos (dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

3.237 $\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=83

$$\frac{3\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)}{bd(1-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[Out] (3*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d
*x]^2])
```

Rubi [A] time = 0.0311037, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$\frac{3\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)}{bd(1-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d
*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{4}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\cos(c+dx)}}\\ &= \frac{3\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1+3m);\frac{1}{6}(5+3m);\cos^2(c+dx)\right)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.109982, size = 82, normalized size = 0.99

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(m-\frac{1}{3}\right);\frac{1}{2}\left(m+\frac{5}{3}\right);\cos^2(c+dx)\right)}{d\left(m-\frac{1}{3}\right)(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/3 + m)/2, (
5/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-1/3 + m)*(b*Cos[c +
d*x])^(4/3)))
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^{m} (b\cos (dx+c))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^{m}}{(b\cos (dx+c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos (dx+c))^{\frac{2}{3}}\cos (dx+c)^{m}}{b^{2}\cos (dx+c)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Integral(cos(c + d*x)**m/(b*cos(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

3.238 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0211971, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= \frac{\int (b\cos(c+dx))^{2/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.022831, size = 58, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(5*b^2*d)
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^2 (b\cos (dx+c))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^2}{(b\cos (dx+c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos (dx+c))^{\frac{2}{3}}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.239 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(2*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0205319, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(2*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= \frac{\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0080266, size = 58, normalized size = 1.

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2bd\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sqrt[Sin[
c + d*x]^2])/(2*b*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(b\cos(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c))^{\frac{2}{3}}}{b^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.240 $\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*
Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0155542, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*
Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0216037, size = 53, normalized size = 0.95

$$\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-4/3),x]
```

```
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sqrt[Sin[
c + d*x]^2])/(d*(b*Cos[c + d*x])^(4/3))
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(1/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c))^{\frac{2}{3}}}{b^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(-4/3), x)
```

3.241 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=55

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

```
[Out] (3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*
Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0267245, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*
Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= b\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx\\ &= \frac{3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0125068, size = 56, normalized size = 1.02

$$\frac{3b\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(7/3))
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(b\cos(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)}{b^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.242 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}$$

```
[Out] (3*b*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*
(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0585996, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*
(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{10/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0632533, size = 58, normalized size = 1.04

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\cot(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int \left(\sec \left(dx + c\right)\right)^{2} \left(b \cos \left(dx + c\right)\right)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(dx + c\right)^{2}}{\left(b \cos \left(dx + c\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b \cos \left(dx + c\right)\right)^{\frac{2}{3}} \sec \left(dx + c\right)^{2}}{b^{2} \cos \left(dx + c\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.243 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{10/3}}$$

```
[Out] (3*b^2*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10
*d*(b*Cos[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0424405, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {16, 2643}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10
*d*(b*Cos[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{13/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.109637, size = 58, normalized size = 1.

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\csc(c+dx)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)}{10d(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int \left(\sec \left(dx+c\right)\right)^{3} \left(b\cos \left(dx+c\right)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(dx+c\right)^{3}}{\left(b\cos \left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\cos \left(dx+c\right)\right)^{\frac{2}{3}}\sec \left(dx+c\right)^{3}}{b^{2}\cos \left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

3.244 $\int (a\cos(e+fx))^m (b\cos(e+fx))^n\,dx$

Optimal. Leaf size=82

$$-\frac{\sin(e+fx)(a\cos(e+fx))^{m+1}(b\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(e+fx)\right)}{af(m+n+1)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*(b*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 +
m + n)/2, (3 + m + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(a*f*(1 + m + n)*Sqr
t[Sin[e + f*x]^2]))
```

Rubi [A] time = 0.0343084, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$-\frac{\sin(e+fx)(a\cos(e+fx))^{m+1}(b\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(e+fx)\right)}{af(m+n+1)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Cos[e + f*x])^n,x]
```

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*(b*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 +
m + n)/2, (3 + m + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(a*f*(1 + m + n)*Sqr
t[Sin[e + f*x]^2]))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx = \left((a\cos(e+fx))^{-n}(b\cos(e+fx))^n\right)\int (a\cos(e+fx))^{m+n}\,dx$$
$$= -\frac{(a\cos(e+fx))^{1+m}(b\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\cos^2\right.}{af(1+m+n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.0847628, size = 77, normalized size = 0.94

$$-\frac{\sqrt{\sin^2(e+fx)}\cot(e+fx)(a\cos(e+fx))^m(b\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(e+fx)\right)}{f(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Cos[e + f*x])^n,x]
```

```
[Out] -(((a*Cos[e + f*x])^m*(b*Cos[e + f*x])^n*Cot[e + f*x]*Hypergeometric2F1[1/2
, (1 + m + n)/2, (3 + m + n)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])/(f*(1
 + m + n)))
```

Maple [F] time = 1.096, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m} \left(b\cos\left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m} \left(b\cos\left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*cos(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\cos\left(fx+e\right)\right)^{m} \left(b\cos\left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(f*x + e))^m*(b*cos(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(e+fx\right)\right)^{m} \left(b\cos\left(e+fx\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*cos(f*x+e))**n,x)
```

```
[Out] Integral((a*cos(e + f*x))**m*(b*cos(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(fx+e))^m (b\cos(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*cos(f*x + e))^n, x)
```

3.245 $\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx$

Optimal. Leaf size=69

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -(((b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2]))
```

Rubi [A] time = 0.0330704, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n,x]
```

```
[Out] -(((b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx &= \frac{\int (b\cos(c+dx))^{2+n}\,dx}{b^2}\\ &= -\frac{(b\cos(c+dx))^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0775167, size = 72, normalized size = 1.04

$$-\frac{\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (3
 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(3 + n)))
```

Maple [F] time = 1.323, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\cos(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \cos(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^n \cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \cos(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

3.246 $\int \cos(c + dx)(b\cos(c + dx))^n\, dx$

Optimal. Leaf size=69

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -(((b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2]))
```

Rubi [A] time = 0.0491827, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {16, 2643}

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n,x]
```

```
[Out] -(((b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^n\,dx &= \frac{\int (b\cos(c+dx))^{1+n}\,dx}{b}\\ &= -\frac{(b\cos(c+dx))^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0540431, size = 70, normalized size = 1.01

$$-\frac{\sqrt{\sin^2(c+dx)}\cos(c+dx)\cot(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (2 +
 n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(2 + n)))
```

Maple [F] time = 1.167, size = 0, normalized size = 0.

$$\int \cos (dx + c) (b \cos (dx + c))^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \cos (dx + c))^{n} \cos (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \cos (dx + c))^{n} \cos (dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*cos(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \cos (c + dx))^{n} \cos (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \cos(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c), x)
```

3.247 $\int (b\cos(c+dx))^n\,dx$

Optimal. Leaf size=69

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -(((b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2]))
```

Rubi [A] time = 0.02375, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n,x]
```

```
[Out] -(((b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^n\,dx = -\frac{(b\cos(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0424394, size = 64, normalized size = 0.93

$$-\frac{\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1 + n)))
```

Maple [F] time = 0.398, size = 0, normalized size = 0.

$$\int \left(b \cos \left(dx + c\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n,x)
```

```
[Out] int((b*cos(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \cos \left(dx + c\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \cos \left(dx + c\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \cos \left(c + dx\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n,x)
```

```
[Out] Integral((b*cos(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \cos \left(dx + c\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n, x)
```

3.248 $\int (b\cos(c+dx))^n \sec(c+dx)\,dx$

Optimal. Leaf size=60

$$-\frac{\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

```
[Out] -(((b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2
]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2]))
```

Rubi [A] time = 0.0302036, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {16, 2643}

$$-\frac{\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*Sec[c + d*x],x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2
]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n\sec(c+dx)\,dx &= b\int (b\cos(c+dx))^{-1+n}\,dx\\ &= -\frac{(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0558222, size = 63, normalized size = 1.05

$$-\frac{b\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x],x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*Hypergeometric2F1[1/2, n/2, (2
+ n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*n))
```

Maple [F] time = 0.711, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \sec(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^n*sec(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \sec(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c))^n \sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(c+dx))^n \sec(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*sec(d*x+c),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c), x)
```

3.249 $\int (b\cos(c+dx))^n \sec^2(c+dx)\,dx$

Optimal. Leaf size=68

$$\frac{b\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (b*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0447606, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$\frac{b\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*Sec[c + d*x]^2,x]
```

```
[Out] (b*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n \sec^2(c+dx)\,dx &= b^2\int (b\cos(c+dx))^{-2+n}\,dx\\ &= \frac{b(b\cos(c+dx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.064868, size = 67, normalized size = 0.99

$$-\frac{b\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x]^2,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)
/2, (1 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-1 + n)))
```

Maple [F] time = 0.658, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (\sec(dx+c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^n*sec(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^n \sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(c+dx))^n \sec^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*sec(d*x+c)**2,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

3.250 $\int (b\cos(c+dx))^n \sec^3(c+dx)\,dx$

Optimal. Leaf size=68

$$\frac{b^2\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (b^2*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0418703, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$\frac{b^2\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*Sec[c + d*x]^3,x]
```

```
[Out] (b^2*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^n \sec^3(c+dx)\,dx = b^3\int (b\cos(c+dx))^{-3+n}\,dx$$
$$= \frac{b^2(b\cos(c+dx))^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0468303, size = 70, normalized size = 1.03

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\sec^2(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x]^3,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2, n/2,
Cos[c + d*x]^2]*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(d*(-2 + n)))
```

Maple [F] time = 0.769, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (\sec(dx+c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^n*sec(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \sec(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^n \sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \sec(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

3.251 $\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx$

Optimal. Leaf size=70

$$\frac{b^3\sin(c+dx)(b\cos(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (b^3*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0420231, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {16, 2643}

$$\frac{b^3\sin(c+dx)(b\cos(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*Sec[c + d*x]^4,x]
```

```
[Out] (b^3*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx &= b^4\int (b\cos(c+dx))^{-4+n}\,dx\\ &= \frac{b^3(b\cos(c+dx))^{-3+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+n);\frac{1}{2}(-1+n);\cos^2(c+dx)\right)\sin(c+dx)}{d(3-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0559101, size = 72, normalized size = 1.03

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\sec^3(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)}{d(n-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x]^4,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 +
 n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(d*(-3 + n)))
```

Maple [F] time = 0.57, size = 0, normalized size = 0.

$$\int \left(b \cos \left(dx+c\right)\right)^{n} \left(\sec \left(dx+c\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*sec(d*x+c)^4,x)
```

```
[Out] int((b*cos(d*x+c))^n*sec(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \cos \left(dx+c\right)\right)^{n} \sec \left(dx+c\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \cos \left(dx+c\right)\right)^{n} \sec \left(dx+c\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \sec(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

3.252 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx$

Optimal. Leaf size=80

$$-\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/
4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c + d*
x]^2])
```

Rubi [A] time = 0.0270646, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$-\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n,x]
```


```
[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/
4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c + d*
x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{5}{2}+n}(c+dx)\,dx$$

$$= -\frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7+2n);\frac{1}{4}(11+2n);\cos^2(c+dx)\right)\sin(}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.140019, size = 80, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)\csc(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(n+\frac{7}{2}\right);\frac{1}{2}\left(n+\frac{11}{2}\right);\cos^2(c+dx)\right)}{d\left(n+\frac{7}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2
, (7/2 + n)/2, (11/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(7/2
+ n)))
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{5}{2}}(b\cos(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (dx+c))^n \cos (dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

3.253 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx$

Optimal. Leaf size=80

$$-\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/
4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rubi [A] time = 0.0284351, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$-\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n,x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/
4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\,dx$$

$$= -\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\cos^2(c+dx)\right)\sin(c+}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.118614, size = 80, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)\csc(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(n+\frac{5}{2}\right);\frac{1}{2}\left(n+\frac{9}{2}\right);\cos^2(c+dx)\right)}{d\left(n+\frac{5}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2
, (5/2 + n)/2, (9/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(5/2 +
 n)))
```

Maple [F] time = 0.223, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{3}{2}}(b\cos(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \cos(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

3.254 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx$

Optimal. Leaf size=80

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/
4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rubi [A] time = 0.0266442, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n,x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/
4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+dx)\,dx$$

$$= -\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\cos^2(c+dx)\right)\sin(c}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0934868, size = 80, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(n+\frac{3}{2}\right);\frac{1}{2}\left(n+\frac{7}{2}\right);\cos^2(c+dx)\right)}{d\left(n+\frac{3}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2
, (3/2 + n)/2, (7/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(3/2 +
 n)))
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int \sqrt{\cos (dx+c)}\,(b\cos (dx+c))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (dx+c))^{n}\sqrt{\cos (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos (dx+c))^{n}\sqrt{\cos (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (c+dx))^{n}\sqrt{\cos (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**n,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \sqrt{\cos(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

3.255 $\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/
4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rubi [A] time = 0.0266147, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/
4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)\,dx\\ &= -\frac{2\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(1+2n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0589562, size = 80, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(n+\frac{1}{2}\right);\frac{1}{2}\left(n+\frac{5}{2}\right);\cos^2(c+dx)\right)}{d\left(n+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2
, (1/2 + n)/2, (5/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1/2 +
 n)))
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n \frac{1}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

3.256 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}$$

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos
[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x
]^2])
```

Rubi [A] time = 0.0285066, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos
[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\,dx$$

$$= \frac{2(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0823002, size = 80, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(n-\frac{1}{2}\right);\frac{1}{2}\left(n+\frac{3}{2}\right);\cos^2(c+dx)\right)}{d\left(n-\frac{1}{2}\right)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(3/2),x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/2 + n)/2, (3/
2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-1/2 + n)*Sqrt[Cos[c +
d*x]]))
```

Maple [F] time = 0.218, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (\cos(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x)
```

```
[Out] int((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n/cos(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

3.257 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos
[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x
]^2])
```

Rubi [A] time = 0.0284384, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos
[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\,dx$$
$$= \frac{2(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0868932, size = 80, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(n-\frac{3}{2}\right);\frac{1}{2}\left(n+\frac{1}{2}\right);\cos^2(c+dx)\right)}{d\left(n-\frac{3}{2}\right)\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(5/2),x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-3/2 + n)/2, (1/
2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-3/2 + n)*Cos[c + d*x]^
(3/2)))
```

Maple [F] time = 0.232, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (\cos(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x)
```

```
[Out] int((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

3.258 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c + d*
x]^2])
```

Rubi [A] time = 0.0279148, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c + d*
x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx)\,dx$$

$$= \frac{2(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0877907, size = 80, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(n-\frac{5}{2}\right);\frac{1}{2}\left(n-\frac{1}{2}\right);\cos^2(c+dx)\right)}{d\left(n-\frac{5}{2}\right)\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(7/2),x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-5/2 + n)/2, (-1
/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-5/2 + n)*Cos[c + d*x]
^(5/2)))
```

Maple [F] time = 0.229, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (\cos(dx+c))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x)
```

```
[Out] int((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

3.259 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-7);\frac{1}{4}(2n-3);\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)}$$

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(7 - 2*n)*Cos[c + d*x]^(7/2)*Sqrt[Sin[c + d*
x]^2])
```

Rubi [A] time = 0.0289639, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-7);\frac{1}{4}(2n-3);\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(7 - 2*n)*Cos[c + d*x]^(7/2)*Sqrt[Sin[c + d*
x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{9}{2}+n}(c+dx)\,dx\\ &= \frac{2(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-7+2n);\frac{1}{4}(-3+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0903199, size = 80, normalized size = 1.

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(n-\frac{7}{2}\right);\frac{1}{2}\left(n-\frac{3}{2}\right);\cos^2(c+dx)\right)}{d\left(n-\frac{7}{2}\right)\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(9/2),x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-7/2 + n)/2, (-3
/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-7/2 + n)*Cos[c + d*x]
^(7/2)))
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (\cos(dx+c))^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x)
```

```
[Out] int((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

3.260 $\int (a\cos(e+fx))^m (b\sec(e+fx))^n\,dx$

Optimal. Leaf size=88

$$-\frac{\sin(e+fx)(a\cos(e+fx))^{m+1}(b\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m-n+1);\frac{1}{2}(m-n+3);\cos^2(e+fx)\right)}{af(m-n+1)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m - n)/2, (3 + m -
n)/2, Cos[e + f*x]^2]*(b*Sec[e + f*x])^n*Sin[e + f*x])/(a*f*(1 + m - n)*Sqr
t[Sin[e + f*x]^2]))
```

Rubi [A] time = 0.070436, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2588, 2643}

$$-\frac{\sin(e+fx)(a\cos(e+fx))^{m+1}(b\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m-n+1);\frac{1}{2}(m-n+3);\cos^2(e+fx)\right)}{af(m-n+1)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Sec[e + f*x])^n,x]
```

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m - n)/2, (3 + m -
n)/2, Cos[e + f*x]^2]*(b*Sec[e + f*x])^n*Sin[e + f*x])/(a*f*(1 + m - n)*Sqr
t[Sin[e + f*x]^2]))
```

Rule 2588

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[(a*b)^IntPart[n]*(a*Sin[e + f*x])^FracPart[n]*(b*Csc
[e + f*x])^FracPart[n], Int[(a*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b
, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx = \left((a\cos(e+fx))^n(b\sec(e+fx))^n\right)\int (a\cos(e+fx))^{m-n}\,dx$$

$$= -\frac{(a\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m-n);\frac{1}{2}(3+m-n);\cos^2(e+fx)\right)(b\sec(}{af(1+m-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 10.4935, size = 89, normalized size = 1.01

$$-\frac{\sin(e+fx)\cos(e+fx)(a\cos(e+fx))^m(b\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m-n+1);\frac{1}{2}(m-n+3);\cos^2(e+fx)\right)}{f(m-n+1)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Sec[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*(a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m - n)/2, (3
 + m - n)/2, Cos[e + f*x]^2]*(b*Sec[e + f*x])^n*Sin[e + f*x])/(f*(1 + m - n
)*Sqrt[Sin[e + f*x]^2]))
```

Maple [F] time = 1.065, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\left(b\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\left(b\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\cos\left(fx+e\right)\right)^{m}\left(b\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(f*x + e))^m*(b*sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(e+fx\right)\right)^{m}\left(b\sec\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*sec(f*x+e))**n,x)
```

```
[Out] Integral((a*cos(e + f*x))**m*(b*sec(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^m \left(b\sec\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*sec(f*x + e))^n, x)
```

3.261 $\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=15

$$\frac{2}{b\sqrt{\csc(a+bx)}}$$

[Out] 2/(b*Sqrt[Csc[a + b*x]])

Rubi [A] time = 0.0246537, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2621, 30}

$$\frac{2}{b\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]

[Out] 2/(b*Sqrt[Csc[a + b*x]])

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \csc(a+bx)\right)}{b}\\ &= \frac{2}{b\sqrt{\csc(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0205176, size = 15, normalized size = 1.

$$\frac{2}{b\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]

[Out] 2/(b*Sqrt[Csc[a + b*x]])

Maple [A] time = 0.042, size = 14, normalized size = 0.9

$$2\frac{1}{b\sqrt{\csc\left(bx+a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^(1/2),x)
```

```
[Out] 2/b/csc(b*x+a)^(1/2)
```

Maxima [A] time = 0.938317, size = 18, normalized size = 1.2

$$\frac{2\sqrt{\sin\left(bx+a\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(sin(b*x + a))/b
```

Fricas [A] time = 1.10671, size = 32, normalized size = 2.13

$$\frac{2\sqrt{\sin\left(bx+a\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(sin(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\cos\left(a+bx\right)\sqrt{\csc\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)*sqrt(csc(a + b*x)), x)
```

Giac [A] time = 1.14225, size = 27, normalized size = 1.8

$$\frac{2\,\mathrm{sgn}\left(\sin\left(bx+a\right)\right)\sqrt{\sin\left(bx+a\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sgn(sin(b*x + a))*sqrt(sin(b*x + a))/b
```

3.262 $\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=17

$$\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/(3*b*Csc[a + b*x]^(3/2))
```

Rubi [A] time = 0.0247542, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2621, 30}

$$\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

```
[Out] 2/(3*b*Csc[a + b*x]^(3/2))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, \csc(a+bx)\right)}{b}\\ &= \frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0244207, size = 17, normalized size = 1.

$$\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

```
[Out] 2/(3*b*Csc[a + b*x]^(3/2))
```

Maple [A] time = 0.029, size = 14, normalized size = 0.8

$$\frac{2}{3b}(\csc(bx+a))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

```
[Out] 2/3/b/csc(b*x+a)^(3/2)
```

Maxima [A] time = 0.943326, size = 18, normalized size = 1.06

$$\frac{2\sin(bx+a)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sin(b*x + a)^(3/2)/b
```

Fricas [A] time = 1.09656, size = 68, normalized size = 4.

$$-\frac{2\left(\cos(bx+a)^2-1\right)}{3b\sqrt{\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(b*x + a)^2 - 1)/(b*sqrt(sin(b*x + a)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)/sqrt(csc(a + b*x)), x)
```

Giac [B] time = 1.31045, size = 43, normalized size = 2.53

$$\frac{2\,\mathrm{sgn}\left(\sin\left(bx+a\right)\right)^{3}\sin\left(bx+a\right)^{\frac{3}{2}}}{3\,b\mathrm{sgn}\left(\mathrm{sgn}\left(\sin\left(bx+a\right)\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sgn(sin(b*x + a))^3*sin(b*x + a)^(3/2)/(b*sgn(sgn(sin(b*x + a)))^2)
```

3.263 $\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=67

$$\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b}$$

```
[Out] (2*Cos[a + b*x])/(3*b*Sqrt[Csc[a + b*x]]) + (4*Sqrt[Csc[a + b*x]]*EllipticF
[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b)
```

Rubi [A] time = 0.0505877, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2628, 3771, 2641}

$$\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*Cos[a + b*x])/(3*b*Sqrt[Csc[a + b*x]]) + (4*Sqrt[Csc[a + b*x]]*EllipticF
[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b)
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx &= \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{2}{3}\int \sqrt{\csc(a+bx)}\,dx\\
&= \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{1}{3}\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx\\
&= \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{4\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{3b}
\end{aligned}$$

Mathematica [A] time = 0.101008, size = 53, normalized size = 0.79

$$\frac{\sqrt{\csc(a+bx)}\left(\sin(2(a+bx))-4\sqrt{\sin(a+bx)}F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(-4*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*
x]] + Sin[2*(a + b*x)]))/(3*b)
```

Maple [A] time = 1.098, size = 88, normalized size = 1.3

$$\frac{1}{\cos(bx+a)b}\left(\frac{2}{3}\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\text{EllipticF}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)+\frac{2\,(\cos(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*csc(b*x+a)^(1/2),x)
```

```
[Out] (2/3*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellip
ticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+2/3*cos(b*x+a)^2*sin(b*x+a))/cos(b*x
+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^2\sqrt{\csc(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2*sqrt(csc(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos(bx+a)^2\sqrt{\csc(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*sqrt(csc(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**2*sqrt(csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^2 \sqrt{\csc(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sqrt(csc(b*x + a)), x)
```

3.264 $\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}} \, dx$

Optimal. Leaf size=67

$$\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}$$

```
[Out] (2*Cos[a + b*x])/(5*b*Csc[a + b*x]^(3/2)) + (4*Sqrt[Csc[a + b*x]]*EllipticE
[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(5*b)
```

Rubi [A] time = 0.0483108, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2628, 3771, 2639}

$$\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*Cos[a + b*x])/(5*b*Csc[a + b*x]^(3/2)) + (4*Sqrt[Csc[a + b*x]]*EllipticE
[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(5*b)
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)} + \frac{2}{5}\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx$$
$$= \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)} + \frac{1}{5}\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx$$
$$= \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{5b}$$

Mathematica [A] time = 0.138767, size = 61, normalized size = 0.91

$$-\frac{2\sqrt{\csc(a+bx)}\left(2\sqrt{\sin(a+bx)}E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)-\sin^2(a+bx)\cos(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (-2*Sqrt[Csc[a + b*x]]*(2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]] - Cos[a + b*x]*Sin[a + b*x]^2))/(5*b)
```

Maple [A] time = 1.122, size = 142, normalized size = 2.1

$$\frac{1}{\cos(bx+a)\,b}\left(-\frac{2\,(\sin(bx+a))^4}{5}+\frac{2\,(\sin(bx+a))^2}{5}-\frac{4}{5}\sqrt{\sin(bx+a)+1}\sqrt{-2\,\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\mathrm{Elliptic}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/csc(b*x+a)^(1/2),x)
```

```
[Out] (-2/5*sin(b*x+a)^4+2/5*sin(b*x+a)^2-4/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)
+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+2
/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellipti
cF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(bx+a\right)^2}{\sqrt{\csc\left(bx+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2\left(a+bx\right)}{\sqrt{\csc\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/sqrt(csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)^2}{\sqrt{\csc\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

3.265 $\int \cos^3(x)\csc^{\frac{9}{2}}(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\csc^{\frac{3}{2}}(x)-\frac{2}{7}\csc^{\frac{7}{2}}(x)$$

```
[Out] (2*Csc[x]^(3/2))/3 - (2*Csc[x]^(7/2))/7
```

Rubi [A] time = 0.0240884, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2621, 14}

$$\frac{2}{3}\csc^{\frac{3}{2}}(x)-\frac{2}{7}\csc^{\frac{7}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Csc[x]^(9/2),x]
```

```
[Out] (2*Csc[x]^(3/2))/3 - (2*Csc[x]^(7/2))/7
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\csc^{\frac{9}{2}}(x)\,dx &= -\operatorname{Subst}\left(\int \sqrt{x}\left(-1+x^2\right)\,dx,x,\csc(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(-\sqrt{x}+x^{5/2}\right)\,dx,x,\csc(x)\right)\\ &= \frac{2}{3}\csc^{\frac{3}{2}}(x)-\frac{2}{7}\csc^{\frac{7}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.0246286, size = 18, normalized size = 0.86

$$\frac{2}{21}\csc^{\frac{3}{2}}(x)\left(7-3\csc^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Csc[x]^(9/2),x]
```

```
[Out] (2*Csc[x]^(3/2)*(7 - 3*Csc[x]^2))/21
```

Maple [A] time = 0.51, size = 14, normalized size = 0.7

$$-\frac{2}{7}(\sin(x))^{-\frac{7}{2}}+\frac{2}{3}(\sin(x))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*csc(x)^(9/2),x)
```

```
[Out] -2/7/sin(x)^(7/2)+2/3/sin(x)^(3/2)
```

Maxima [A] time = 0.963319, size = 18, normalized size = 0.86

$$\frac{2}{3\sin(x)^{\frac{3}{2}}}-\frac{2}{7\sin(x)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*csc(x)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/3/sin(x)^(3/2) - 2/7/sin(x)^(7/2)
```

Fricas [A] time = 1.02241, size = 72, normalized size = 3.43

$$\frac{2\left(7\cos(x)^{2}-4\right)}{21\left(\cos(x)^{2}-1\right)\sin(x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*csc(x)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/21*(7*cos(x)^2 - 4)/((cos(x)^2 - 1)*sin(x)^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*csc(x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13093, size = 23, normalized size = 1.1

$$\frac{2\left(7\sin(x)^{2}-3\right)\operatorname{sgn}(\sin(x))}{21\sin(x)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*csc(x)^(9/2),x, algorithm="giac")
```

```
[Out] 2/21*(7*sin(x)^2 - 3)*sgn(sin(x))/sin(x)^(7/2)
```

3.266 $\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=33

$$\frac{2}{b\sqrt{\csc(a+bx)}}-\frac{2}{5b\csc^{\frac{5}{2}}(a+bx)}$$

```
[Out] -2/(5*b*Csc[a + b*x]^(5/2)) + 2/(b*Sqrt[Csc[a + b*x]])
```

Rubi [A] time = 0.0326656, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2621, 14}

$$\frac{2}{b\sqrt{\csc(a+bx)}}-\frac{2}{5b\csc^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sqrt[Csc[a + b*x]],x]
```

```
[Out] -2/(5*b*Csc[a + b*x]^(5/2)) + 2/(b*Sqrt[Csc[a + b*x]])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{-1+x^2}{x^{7/2}}\,dx,x,\csc(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{1}{x^{7/2}}+\frac{1}{x^{3/2}}\right)dx,x,\csc(a+bx)\right)}{b}\\ &= -\frac{2}{5b\csc^{\frac{5}{2}}(a+bx)}+\frac{2}{b\sqrt{\csc(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0576229, size = 27, normalized size = 0.82

$$\frac{\cos(2(a+bx))+9}{5b\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sqrt[Csc[a + b*x]],x]

[Out] (9 + Cos[2*(a + b*x)])/(5*b*Sqrt[Csc[a + b*x]])
```

Maple [A] time = 0.565, size = 26, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{2}{5}\left(\sin\left(bx+a\right)\right)^{\frac{5}{2}}+2\sqrt{\sin\left(bx+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*csc(b*x+a)^(1/2),x)

[Out] (-2/5*sin(b*x+a)^(5/2)+2*sin(b*x+a)^(1/2))/b
```

Maxima [A] time = 0.95676, size = 34, normalized size = 1.03

$$\frac{2\left(\frac{5}{\sin\left(bx+a\right)^{2}}-1\right)\sin\left(bx+a\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*csc(b*x+a)^(1/2),x, algorithm="maxima")

[Out] 2/5*(5/sin(b*x + a)^2 - 1)*sin(b*x + a)^(5/2)/b
```

Fricas [A] time = 1.07112, size = 63, normalized size = 1.91

$$\frac{2\left(\cos\left(bx+a\right)^{2}+4\right)\sqrt{\sin\left(bx+a\right)}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*csc(b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2/5*(cos(b*x + a)^2 + 4)*sqrt(sin(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*csc(b*x+a)**(1/2),x)

[Out] Timed out
```

Giac [A] time = 1.18348, size = 42, normalized size = 1.27

$$-\frac{2\left(\sin\left(bx+a\right)^{\frac{5}{2}}-5\sqrt{\sin\left(bx+a\right)}\right)\mathrm{sgn}\left(\sin\left(bx+a\right)\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*(sin(b*x + a)^(5/2) - 5*sqrt(sin(b*x + a)))*sgn(sin(b*x + a))/b
```

3.267 $\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}} dx$

Optimal. Leaf size=35

$$\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2}{7b\csc^{\frac{7}{2}}(a+bx)}$$

```
[Out] -2/(7*b*Csc[a + b*x]^(7/2)) + 2/(3*b*Csc[a + b*x]^(3/2))
```

Rubi [A] time = 0.0327947, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2621, 14}

$$\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2}{7b\csc^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sqrt[Csc[a + b*x]],x]
```

```
[Out] -2/(7*b*Csc[a + b*x]^(7/2)) + 2/(3*b*Csc[a + b*x]^(3/2))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}} dx &= -\frac{\text{Subst}\left(\int \frac{-1+x^2}{x^{9/2}} dx, x, \csc(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \left(-\frac{1}{x^{9/2}} + \frac{1}{x^{5/2}}\right) dx, x, \csc(a+bx)\right)}{b} \\
&= -\frac{2}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}
\end{aligned}$$

Mathematica [A] time = 0.0637448, size = 29, normalized size = 0.83

$$\frac{2\left(7\csc^2(a+bx)-3\right)}{21b\csc^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*(-3 + 7*Csc[a + b*x]^2))/(21*b*Csc[a + b*x]^(7/2))
```

Maple [A] time = 0.53, size = 26, normalized size = 0.7

$$\frac{1}{b}\left(-\frac{2}{7}\left(\sin\left(bx+a\right)\right)^{\frac{7}{2}}+\frac{2}{3}\left(\sin\left(bx+a\right)\right)^{\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/csc(b*x+a)^(1/2),x)
```

```
[Out] (-2/7*sin(b*x+a)^(7/2)+2/3*sin(b*x+a)^(3/2))/b
```

Maxima [A] time = 0.998336, size = 34, normalized size = 0.97

$$\frac{2\left(\frac{7}{\sin(bx+a)^2}-3\right)\sin\left(bx+a\right)^{\frac{7}{2}}}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(7/sin(b*x + a)^2 - 3)*sin(b*x + a)^(7/2)/b
```

Fricas [A] time = 1.10725, size = 95, normalized size = 2.71

$$-\frac{2\left(3\,\cos\left(bx+a\right)^{4}+\cos\left(bx+a\right)^{2}-4\right)}{21\,b\sqrt{\sin\left(bx+a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21*(3*cos(b*x + a)^4 + cos(b*x + a)^2 - 4)/(b*sqrt(sin(b*x + a)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/csc(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25266, size = 34, normalized size = 0.97

$$\frac{2\left(\frac{7}{\sin(bx+a)^{2}} - 3\right)\sin\left(bx + a\right)^{\frac{7}{2}}}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*(7/sin(b*x + a)^2 - 3)*sin(b*x + a)^(7/2)/b
```

3.268 $\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=92

$$\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}+\frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}}+\frac{8\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{7b}$$

```
[Out] (4*Cos[a + b*x])/(7*b*Sqrt[Csc[a + b*x]]) + (2*Cos[a + b*x]^3)/(7*b*Sqrt[Cs
c[a + b*x]]) + (8*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[
Sin[a + b*x]])/(7*b)
```

Rubi [A] time = 0.0801452, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2628, 3771, 2641}

$$\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}+\frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}}+\frac{8\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (4*Cos[a + b*x])/(7*b*Sqrt[Csc[a + b*x]]) + (2*Cos[a + b*x]^3)/(7*b*Sqrt[Cs
c[a + b*x]]) + (8*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[
Sin[a + b*x]])/(7*b)
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{6}{7}\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx$$
$$= \frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{4}{7}\int \sqrt{\csc(a+bx)}\,dx$$
$$= \frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{1}{7}\left(4\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a+bx)}}$$
$$= \frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{8\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{7b}$$

Mathematica [A] time = 0.147228, size = 63, normalized size = 0.68

$$\frac{\sqrt{\csc(a+bx)}\left(10\sin(2(a+bx))+\sin(4(a+bx))-32\sqrt{\sin(a+bx)}F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{28b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(-32*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b
*x]] + 10*Sin[2*(a + b*x)] + Sin[4*(a + b*x)]))/(28*b)
```

Maple [A] time = 1.016, size = 100, normalized size = 1.1

$$\frac{1}{\cos(bx+a)\,b}\left(\frac{2\,(\sin(bx+a))^5}{7} - \frac{8\,(\sin(bx+a))^3}{7} + \frac{6\,\sin(bx+a)}{7} + \frac{4}{7}\sqrt{\sin(bx+a)+1}\sqrt{-2\,\sin(bx+a)+2}\sqrt{-\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*csc(b*x+a)^(1/2),x)
```

```
[Out] (2/7*sin(b*x+a)^5-8/7*sin(b*x+a)^3+6/7*sin(b*x+a)+4/7*(sin(b*x+a)+1)^(1/2)*
(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),
1/2*2^(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^4\sqrt{\csc(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^4*sqrt(csc(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos(bx+a)^4\sqrt{\csc(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^4*sqrt(csc(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*csc(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos{(bx+a)}^4 \sqrt{\csc{(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^4*sqrt(csc(b*x + a)), x)
```

3.269 $\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=92

$$\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)}+\frac{8\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b}$$

```
[Out] (4*Cos[a + b*x])/(15*b*Csc[a + b*x]^(3/2)) + (2*Cos[a + b*x]^3)/(9*b*Csc[a
+ b*x]^(3/2)) + (8*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt
[Sin[a + b*x]])/(15*b)
```

Rubi [A] time = 0.0788764, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2628, 3771, 2639}

$$\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)}+\frac{8\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (4*Cos[a + b*x])/(15*b*Csc[a + b*x]^(3/2)) + (2*Cos[a + b*x]^3)/(9*b*Csc[a
+ b*x]^(3/2)) + (8*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt
[Sin[a + b*x]])/(15*b)
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)} + \frac{2}{3}\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$
$$= \frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)} + \frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)} + \frac{4}{15}\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx$$
$$= \frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)} + \frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)} + \frac{1}{15}\left(4\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx$$
$$= \frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)} + \frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)} + \frac{8\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{15b}$$

Mathematica [A] time = 0.412331, size = 63, normalized size = 0.68

$$\frac{39\cos(a+bx)+5\cos(3(a+bx))-\frac{48E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)}{\sin^{\frac{3}{2}}(a+bx)}}{90b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (39*Cos[a + b*x] + 5*Cos[3*(a + b*x)] - (48*EllipticE[(-2*a + Pi - 2*b*x)/4
, 2])/Sin[a + b*x]^(3/2))/(90*b*Csc[a + b*x]^(3/2))
```

Maple [A] time = 1.095, size = 152, normalized size = 1.7

$$\frac{1}{\cos(bx+a)\,b}\left(-\frac{2\,(\cos(bx+a))^6}{9}-\frac{8}{15}\sqrt{\sin(bx+a)+1}\sqrt{-2\,\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\mathrm{EllipticE}\left(\sqrt{\sin(bx+a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/csc(b*x+a)^(1/2),x)
```

```
[Out] (-2/9*cos(b*x+a)^6-8/15*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(
b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+4/15*(sin(b*x+a)+
1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+
1)^(1/2),1/2*2^(1/2))-2/45*cos(b*x+a)^4+4/15*cos(b*x+a)^2)/cos(b*x+a)/sin(b
*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^4}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(bx+a\right)^{4}}{\sqrt{\csc\left(bx+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{4}\left(a+bx\right)}{\sqrt{\csc\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**4/sqrt(csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)^{4}}{\sqrt{\csc\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

3.270 $\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx$

Optimal. Leaf size=10

$$-\frac{3}{4}\csc^{\frac{4}{3}}(x)$$

```
[Out] (-3*Csc[x]^(4/3))/4
```

Rubi [A] time = 0.0165183, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2621, 30}

$$-\frac{3}{4}\csc^{\frac{4}{3}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[x]^(7/3),x]

[Out] (-3*Csc[x]^(4/3))/4
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx &= -\operatorname{Subst}\left(\int \sqrt[3]{x}\,dx, x, \csc(x)\right)\\ &= -\frac{3}{4}\csc^{\frac{4}{3}}(x)\end{aligned}$$

Mathematica [A] time = 0.0076398, size = 10, normalized size = 1.

$$-\frac{3}{4}\csc^{\frac{4}{3}}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[x]^(7/3),x]

[Out] (-3*Csc[x]^(4/3))/4
```

Maple [A] time = 0.026, size = 7, normalized size = 0.7

$$-\frac{3}{4}\left(\csc\left(x\right)\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(x)^(7/3),x)
```

```
[Out] -3/4*csc(x)^(4/3)
```

Maxima [A] time = 0.962924, size = 8, normalized size = 0.8

$$-\frac{3}{4\,\sin\left(x\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^(7/3),x, algorithm="maxima")
```

```
[Out] -3/4/sin(x)^(4/3)
```

Fricas [A] time = 1.04112, size = 26, normalized size = 2.6

$$-\frac{3}{4\,\sin\left(x\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^(7/3),x, algorithm="fricas")
```

```
[Out] -3/4/sin(x)^(4/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)**(7/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14886, size = 8, normalized size = 0.8

$$-\frac{3}{4\,\sin\left(x\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^(7/3),x, algorithm="giac")
```

```
[Out] -3/4/sin(x)^(4/3)
```

3.271 $\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}-\frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

```
[Out] -(ArcTan[Sqrt[Csc[a + b*x]]]/b) + ArcTanh[Sqrt[Csc[a + b*x]]]/b
```

Rubi [A] time = 0.0278043, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2621, 329, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}-\frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x],x]
```

```
[Out] -(ArcTan[Sqrt[Csc[a + b*x]]]/b) + ArcTanh[Sqrt[Csc[a + b*x]]]/b
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{b}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b}$$
$$= -\frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b} + \frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

Mathematica [A] time = 0.0262676, size = 47, normalized size = 1.47

$$\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\left(\tan^{-1}\left(\sqrt{\sin(a+bx)}\right)+\tanh^{-1}\left(\sqrt{\sin(a+bx)}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x],x]
```

```
[Out] ((ArcTan[Sqrt[Sin[a + b*x]]] + ArcTanh[Sqrt[Sin[a + b*x]]])*Sqrt[Csc[a + b*
x]]*Sqrt[Sin[a + b*x]])/b
```

Maple [A] time = 0.471, size = 28, normalized size = 0.9

$$\frac{1}{b}\text{Artanh}\left(\sqrt{\sin(bx+a)}\right)+\frac{1}{b}\arctan\left(\sqrt{\sin(bx+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^(1/2)*sec(b*x+a),x)
```

```
[Out] 1/b*arctanh(sin(b*x+a)^(1/2))+1/b*arctan(sin(b*x+a)^(1/2))
```

Maxima [A] time = 1.50672, size = 55, normalized size = 1.72

$$-\frac{2\arctan\left(\frac{1}{\sqrt{\sin(bx+a)}}\right)-\log\left(\frac{1}{\sqrt{\sin(bx+a)}}+1\right)+\log\left(\frac{1}{\sqrt{\sin(bx+a)}}-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*arctan(1/sqrt(sin(b*x + a))) - log(1/sqrt(sin(b*x + a)) + 1) + log(
1/sqrt(sin(b*x + a)) - 1))/b
```

Fricas [B] time = 1.27863, size = 274, normalized size = 8.56

$$\frac{2\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin(bx+a)}}\right)+\log\left(\frac{\cos(bx+a)^2+\frac{4\left(\cos(bx+a)^2-\sin(bx+a)-1\right)}{\sqrt{\sin(bx+a)}}-6\sin(bx+a)-2}{\cos(bx+a)^2+2\sin(bx+a)-2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a))) + log((cos(b*x + a
)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) - 1)/sqrt(sin(b*x + a)) - 6*sin(b*x
+ a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x + a) - 2)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\csc(a+bx)}\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**(1/2)*sec(b*x+a),x)
```

```
[Out] Integral(sqrt(csc(a + b*x))*sec(a + b*x), x)
```

Giac [A] time = 1.20699, size = 66, normalized size = 2.06

$$\frac{\left(2\arctan\left(\sqrt{\sin(bx+a)}\right)+\log\left(\sqrt{\sin(bx+a)}+1\right)-\log\left(\left|\sqrt{\sin(bx+a)}-1\right|\right)\right)\mathrm{sgn}(\sin(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*arctan(sqrt(sin(b*x + a))) + log(sqrt(sin(b*x + a)) + 1) - log(abs(s
qrt(sin(b*x + a)) - 1)))*sgn(sin(b*x + a))/b
```

3.272 $\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

```
[Out] ArcTan[Sqrt[Csc[a + b*x]]]/b + ArcTanh[Sqrt[Csc[a + b*x]]]/b
```

Rubi [A] time = 0.0287734, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2621, 329, 212, 206, 203}

$$\frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

```
[Out] ArcTan[Sqrt[Csc[a + b*x]]]/b + ArcTanh[Sqrt[Csc[a + b*x]]]/b
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \csc(a+bx)\right)}{b}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b}$$
$$= \frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b} + \frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

Mathematica [A] time = 0.0338167, size = 50, normalized size = 1.61

$$-\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\left(\tan^{-1}\left(\sqrt{\sin(a+bx)}\right) - \tanh^{-1}\left(\sqrt{\sin(a+bx)}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

```
[Out] -(((ArcTan[Sqrt[Sin[a + b*x]]] - ArcTanh[Sqrt[Sin[a + b*x]]])*Sqrt[Csc[a +
b*x]]*Sqrt[Sin[a + b*x]])/b)
```

Maple [A] time = 1.172, size = 48, normalized size = 1.6

$$-\frac{1}{2b}\ln\left(\sqrt{\sin(bx+a)}-1\right) + \frac{1}{2b}\ln\left(\sqrt{\sin(bx+a)}+1\right) - \frac{1}{b}\arctan\left(\sqrt{\sin(bx+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/csc(b*x+a)^(1/2),x)
```

```
[Out] -1/2/b*ln(sin(b*x+a)^(1/2)-1)+1/2/b*ln(sin(b*x+a)^(1/2)+1)-1/b*arctan(sin(b
*x+a)^(1/2))
```

Maxima [A] time = 1.48677, size = 55, normalized size = 1.77

$$\frac{2\arctan\left(\frac{1}{\sqrt{\sin(bx+a)}}\right) + \log\left(\frac{1}{\sqrt{\sin(bx+a)}}+1\right) - \log\left(\frac{1}{\sqrt{\sin(bx+a)}}-1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arctan(1/sqrt(sin(b*x + a))) + log(1/sqrt(sin(b*x + a)) + 1) - log(1
/sqrt(sin(b*x + a)) - 1))/b
```

Fricas [B] time = 1.31492, size = 275, normalized size = 8.87

$$-\frac{2\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin(bx+a)}}\right)-\log\left(\frac{\cos(bx+a)^2+\frac{4\left(\cos(bx+a)^2-\sin(bx+a)-1\right)}{\sqrt{\sin(bx+a)}}-6\sin(bx+a)-2}{\cos(bx+a)^2+2\sin(bx+a)-2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a))) - log((cos(b*x +
a)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) - 1)/sqrt(sin(b*x + a)) - 6*sin(b*x
 + a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x + a) - 2)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(sec(a + b*x)/sqrt(csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)/sqrt(csc(b*x + a)), x)
```

3.273 $\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx$

Optimal. Leaf size=61

$$\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}+\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

```
[Out] Sec[a + b*x]/(b*Sqrt[Csc[a + b*x]]) + (Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi
/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b
```

Rubi [A] time = 0.0484402, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2626, 3771, 2641}

$$\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}+\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^2,x]
```

```
[Out] Sec[a + b*x]/(b*Sqrt[Csc[a + b*x]]) + (Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi
/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx&=\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}+\frac{1}{2}\int \sqrt{\csc(a+bx)}\,dx\\&=\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}+\frac{1}{2}\left(\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx\\&=\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}+\frac{\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.147641, size = 49, normalized size = 0.8

$$\frac{\sec(a+bx)+\frac{F\left(\frac{1}{4}(2a+2bx-\pi)\middle|2\right)}{\sqrt{\sin(a+bx)}}}{b\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^2,x]
```

```
[Out] (Sec[a + b*x] + EllipticF[(2*a - Pi + 2*b*x)/4, 2]/Sqrt[Sin[a + b*x]])/(b*S
qrt[Csc[a + b*x]])
```

Maple [A] time = 1.664, size = 123, normalized size = 2.

$$\frac{1}{2\cos(bx+a)b}\sqrt{(\cos(bx+a))^2\sin(bx+a)}\left(\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\text{EllipticF}\left(\sqrt{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x)
```

```
[Out] 1/2*(cos(b*x+a)^2*sin(b*x+a))^(1/2)*((sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)
^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+2*si
n(b*x+a))/(-sin(b*x+a)*(sin(b*x+a)-1)*(sin(b*x+a)+1))^(1/2)/cos(b*x+a)/sin(
b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\csc(bx+a)}\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csc(b*x + a))*sec(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\csc(bx+a)}\sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(csc(b*x + a))*sec(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**(1/2)*sec(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(csc(a + b*x))*sec(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\csc(bx+a)}\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(csc(b*x + a))*sec(b*x + a)^2, x)
```

3.274 $\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

```
[Out] Sec[a + b*x]/(b*Csc[a + b*x]^(3/2)) - (Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi
/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b
```

Rubi [A] time = 0.0477514, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2626, 3771, 2639}

$$\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

```
[Out] Sec[a + b*x]/(b*Csc[a + b*x]^(3/2)) - (Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi
/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= \frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)} - \frac{1}{2}\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx \\ &= \frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)} - \frac{1}{2}\left(\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx \\ &= \frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)} - \frac{\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.14791, size = 54, normalized size = 0.87

$$\frac{\sqrt{\csc(a+bx)}\left(\sin(a+bx)\tan(a+bx)+\sqrt{\sin(a+bx)}E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]
 + Sin[a + b*x]*Tan[a + b*x]))/b
```

Maple [B] time = 1.586, size = 177, normalized size = 2.9

$$\frac{1}{2\cos(bx+a)\,b}\sqrt{(\cos(bx+a))^2\sin(bx+a)}\left(2\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\mathrm{EllipticE}\left(\sqrt{\sin(b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/csc(b*x+a)^(1/2),x)
```

```
[Out] 1/2*(cos(b*x+a)^2*sin(b*x+a))^(1/2)*(2*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+
2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-(s
in(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((s
in(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2+2)/(-sin(b*x+a)*(sin(b*x+a)-
1)*(sin(b*x+a)+1))^(1/2)/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)^2}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(bx+a\right)^{2}}{\sqrt{\csc\left(bx+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}\left(a+bx\right)}{\sqrt{\csc\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(sec(a + b*x)**2/sqrt(csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(bx+a\right)^{2}}{\sqrt{\csc\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

3.275 $\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx$

Optimal. Leaf size=62

$$-\frac{3\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}}+\frac{3\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}$$

```
[Out] (-3*ArcTan[Sqrt[Csc[a + b*x]]])/(4*b) + (3*ArcTanh[Sqrt[Csc[a + b*x]]])/(4*
b) + Sec[a + b*x]^2/(2*b*Sqrt[Csc[a + b*x]])
```

Rubi [A] time = 0.0532984, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2621, 288, 329, 298, 203, 206}

$$-\frac{3\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}}+\frac{3\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sqrt[Csc[a + b*x]]])/(4*b) + (3*ArcTanh[Sqrt[Csc[a + b*x]]])/(4*
b) + Sec[a + b*x]^2/(2*b*Sqrt[Csc[a + b*x]])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^{5/2}}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{b}\\
&= \frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{4b}\\
&= \frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\csc(a+bx)}\right)}{2b}\\
&= \frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{4b}\\
&= -\frac{3\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b} + \frac{3\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b} + \frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.113689, size = 73, normalized size = 1.18

$$\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\left(2\sqrt{\sin(a+bx)}\sec^2(a+bx)+3\left(\tan^{-1}\left(\sqrt{\sin(a+bx)}\right)+\tanh^{-1}\left(\sqrt{\sin(a+bx)}\right)\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^3,x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(3*(ArcTan[Sqrt[Sin[a + b*x]]] + ArcTanh[Sqrt[Sin[a + b
*x]]]) + 2*Sec[a + b*x]^2*Sqrt[Sin[a + b*x]])*Sqrt[Sin[a + b*x]])/(4*b)
```

Maple [A] time = 1.51, size = 73, normalized size = 1.2

$$\frac{1}{8\,(\cos(bx+a))^2\,b}\left(-\left(-3\,\ln\left(\sqrt{\sin(bx+a)}+1\right)+3\,\ln\left(\sqrt{\sin(bx+a)}-1\right)-6\,\arctan\left(\sqrt{\sin(bx+a)}\right)\right)(\cos(bx+a)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x)
```

```
[Out] 1/8*(-(-3*ln(sin(b*x+a)^(1/2)+1)+3*ln(sin(b*x+a)^(1/2)-1)-6*arctan(sin(b*x+
a)^(1/2)))*cos(b*x+a)^2+4*sin(b*x+a)^(1/2))/cos(b*x+a)^2/b
```

Maxima [A] time = 1.4409, size = 88, normalized size = 1.42

$$\frac{\frac{4}{\left(\frac{1}{\sin(bx+a)^2}-1\right)\sin(bx+a)^{\frac{3}{2}}} - 6\arctan\left(\frac{1}{\sqrt{\sin(bx+a)}}\right) + 3\log\left(\frac{1}{\sqrt{\sin(bx+a)}}+1\right) - 3\log\left(\frac{1}{\sqrt{\sin(bx+a)}}-1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4/((1/sin(b*x + a)^2 - 1)*sin(b*x + a)^(3/2)) - 6*arctan(1/sqrt(sin(b*
x + a))) + 3*log(1/sqrt(sin(b*x + a)) + 1) - 3*log(1/sqrt(sin(b*x + a)) - 1
))/b
```

Fricas [B] time = 1.29247, size = 373, normalized size = 6.02

$$\frac{6\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin(bx+a)}}\right)\cos(bx+a)^2 + 3\cos(bx+a)^2\log\left(\frac{\cos(bx+a)^2+\frac{4\left(\cos(bx+a)^2-\sin(bx+a)-1\right)}{\sqrt{\sin(bx+a)}}-6\sin(bx+a)-2}{\cos(bx+a)^2+2\sin(bx+a)-2}\right) + 8\sqrt{\sin(bx+a)}}{16b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a)))*cos(b*x + a)^2 +
3*cos(b*x + a)^2*log((cos(b*x + a)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) - 1
)/sqrt(sin(b*x + a)) - 6*sin(b*x + a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x + a)
 - 2)) + 8*sqrt(sin(b*x + a)))/(b*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**(1/2)*sec(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(csc(a + b*x))*sec(a + b*x)**3, x)
```

Giac [A] time = 1.26178, size = 99, normalized size = 1.6

$$-\frac{\left(\frac{4\sqrt{\sin(bx+a)}}{\sin(bx+a)^2-1} - 6\arctan\left(\sqrt{\sin(bx+a)}\right) - 3\log\left(\sqrt{\sin(bx+a)}+1\right) + 3\log\left(\left|\sqrt{\sin(bx+a)}-1\right|\right)\right)\operatorname{sgn}(\sin(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*sqrt(sin(b*x + a))/(sin(b*x + a)^2 - 1) - 6*arctan(sqrt(sin(b*x + a
))) - 3*log(sqrt(sin(b*x + a)) + 1) + 3*log(abs(sqrt(sin(b*x + a)) - 1)))*s
gn(sin(b*x + a))/b
```

3.276 $\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}+\frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}$$

```
[Out] ArcTan[Sqrt[Csc[a + b*x]]]/(4*b) + ArcTanh[Sqrt[Csc[a + b*x]]]/(4*b) + Sec[
a + b*x]^2/(2*b*Csc[a + b*x]^(3/2))
```

Rubi [A] time = 0.0463736, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2621, 288, 329, 212, 206, 203}

$$\frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}+\frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3/Sqrt[Csc[a + b*x]],x]
```

```
[Out] ArcTan[Sqrt[Csc[a + b*x]]]/(4*b) + ArcTanh[Sqrt[Csc[a + b*x]]]/(4*b) + Sec[
a + b*x]^2/(2*b*Csc[a + b*x]^(3/2))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^{3/2}}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{b}\\
&= \frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx, x, \csc(a+bx)\right)}{4b}\\
&= \frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\csc(a+bx)}\right)}{2b}\\
&= \frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{4b}\\
&= \frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b} + \frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b} + \frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}
\end{aligned}
$$

Mathematica [C] time = 0.0312675, size = 33, normalized size = 0.53

$$
\frac{2\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};\sin^2(a+bx)\right)}{3b\csc^{\frac{3}{2}}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 2, 7/4, Sin[a + b*x]^2])/(3*b*Csc[a + b*x]^(3/2))
```

Maple [A] time = 1.461, size = 71, normalized size = 1.2

$$
\frac{1}{8\,(\cos(bx+a))^2 b}\left(-\left(\ln\left(\sqrt{\sin(bx+a)}-1\right)+2\,\arctan\left(\sqrt{\sin(bx+a)}\right)-\ln\left(\sqrt{\sin(bx+a)}+1\right)\right)(\cos(bx+a))^2+4\,(\mathrm{si}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/csc(b*x+a)^(1/2),x)
```

```
[Out] 1/8*(-(ln(sin(b*x+a)^(1/2)-1)+2*arctan(sin(b*x+a)^(1/2))-ln(sin(b*x+a)^(1/2
)+1))*cos(b*x+a)^2+4*sin(b*x+a)^(3/2))/cos(b*x+a)^2/b
```

Maxima [A] time = 1.45519, size = 85, normalized size = 1.37

$$\frac{\frac{4}{\left(\frac{1}{\sin(bx+a)^2}-1\right)\sqrt{\sin(bx+a)}}+2\arctan\left(\frac{1}{\sqrt{\sin(bx+a)}}\right)+\log\left(\frac{1}{\sqrt{\sin(bx+a)}}+1\right)-\log\left(\frac{1}{\sqrt{\sin(bx+a)}}-1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(4/((1/sin(b*x + a)^2 - 1)*sqrt(sin(b*x + a))) + 2*arctan(1/sqrt(sin(b*
x + a))) + log(1/sqrt(sin(b*x + a)) + 1) - log(1/sqrt(sin(b*x + a)) - 1))/b
```

Fricas [B] time = 1.30267, size = 400, normalized size = 6.45

$$-\frac{2\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin(bx+a)}}\right)\cos(bx+a)^2-\cos(bx+a)^2\log\left(\frac{\cos(bx+a)^2+\frac{4\left(\cos(bx+a)^2-\sin(bx+a)-1\right)}{\sqrt{\sin(bx+a)}}-6\sin(bx+a)-2}{\cos(bx+a)^2+2\sin(bx+a)-2}\right)+\frac{8\left(\cos(bx+a)^2-1\right)}{\sqrt{\sin(bx+a)}}}{16b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(2*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a)))*cos(b*x + a)^2 -
 cos(b*x + a)^2*log((cos(b*x + a)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) - 1)
/sqrt(sin(b*x + a)) - 6*sin(b*x + a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x + a)
- 2)) + 8*(cos(b*x + a)^2 - 1)/sqrt(sin(b*x + a)))/(b*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(sec(a + b*x)**3/sqrt(csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)^3}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^3/sqrt(csc(b*x + a)), x)
```

3.277 $\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx$

Optimal. Leaf size=92

$$\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}}+\frac{5\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{6b}$$

```
[Out] (5*Sec[a + b*x])/(6*b*Sqrt[Csc[a + b*x]]) + Sec[a + b*x]^3/(3*b*Sqrt[Csc[a
+ b*x]]) + (5*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[
a + b*x]])/(6*b)
```

Rubi [A] time = 0.0813144, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2626, 3771, 2641}

$$\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}}+\frac{5\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^4,x]
```

```
[Out] (5*Sec[a + b*x])/(6*b*Sqrt[Csc[a + b*x]]) + Sec[a + b*x]^3/(3*b*Sqrt[Csc[a
+ b*x]]) + (5*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[
a + b*x]])/(6*b)
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx = \frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{5}{6}\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx$$
$$= \frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}} + \frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{5}{12}\int \sqrt{\csc(a+bx)}\,dx$$
$$= \frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}} + \frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{1}{12}\left(5\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a}$$
$$= \frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}} + \frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{5\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a}}{6b}$$

Mathematica [A] time = 0.378396, size = 64, normalized size = 0.7

$$\frac{\sqrt{\csc(a+bx)}\left(\tan(a+bx)\left(2\sec^2(a+bx)+5\right)-5\sqrt{\sin(a+bx)}F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^4,x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(-5*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*
x]] + (5 + 2*Sec[a + b*x]^2)*Tan[a + b*x]))/(6*b)
```

Maple [A] time = 1.954, size = 168, normalized size = 1.8

$$-\frac{1}{(12\sin(bx+a)-12)(\sin(bx+a)+1)\cos(bx+a)\,b}\sqrt{(\cos(bx+a))^2\sin(bx+a)}\left(5\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x)
```

```
[Out] -1/12*(cos(b*x+a)^2*sin(b*x+a))^(1/2)*(5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a
)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*
cos(b*x+a)^2+10*cos(b*x+a)^2*sin(b*x+a)+4*sin(b*x+a))/(sin(b*x+a)-1)/(sin(b
*x+a)+1)/(-sin(b*x+a)*(sin(b*x+a)-1)*(sin(b*x+a)+1))^(1/2)/cos(b*x+a)/sin(b
*x+a)^(1/2)/b
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\csc(bx+a)}\sec(bx+a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(csc(b*x + a))*sec(b*x + a)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**(1/2)*sec(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\csc(bx+a)}\sec(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(csc(b*x + a))*sec(b*x + a)^4, x)
```

3.278 $\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=92

$$\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}+\frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{2b}$$

```
[Out] Sec[a + b*x]/(2*b*Csc[a + b*x]^(3/2)) + Sec[a + b*x]^3/(3*b*Csc[a + b*x]^(3
/2)) - (Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*
x]])/(2*b)
```

Rubi [A] time = 0.0803564, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2626, 3771, 2639}

$$\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}+\frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

```
[Out] Sec[a + b*x]/(2*b*Csc[a + b*x]^(3/2)) + Sec[a + b*x]^3/(3*b*Csc[a + b*x]^(3
/2)) - (Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*
x]])/(2*b)
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{1}{2}\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$
$$= \frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)} + \frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{1}{4}\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx$$
$$= \frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)} + \frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{1}{4}\left(\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx$$
$$= \frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)} + \frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{2b}$$

Mathematica [A] time = 0.241992, size = 76, normalized size = 0.83

$$\frac{\cos(a+bx)\sqrt{\csc(a+bx)}\left(2\sec^4(a+bx)+\sec^2(a+bx)+3\sqrt{\sin(a+bx)}\sec(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)-3\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (Cos[a + b*x]*Sqrt[Csc[a + b*x]]*(-3 + Sec[a + b*x]^2 + 2*Sec[a + b*x]^4 +
3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sec[a + b*x]*Sqrt[Sin[a + b*x]]))/(6*
b)
```

Maple [A] time = 2.687, size = 160, normalized size = 1.7

$$\frac{1}{12\,(\cos(bx+a))^3\,b}\left(6\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\mathrm{EllipticE}\left(\sqrt{\sin(bx+a)+1},1/2\sqrt{2}\right)(\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/csc(b*x+a)^(1/2),x)
```

```
[Out] 1/12/sin(b*x+a)^(1/2)/cos(b*x+a)^3*(6*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2
)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos
(b*x+a)^2-3*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2
)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)^2-6*cos(b*x+a)^4+2
*cos(b*x+a)^2+4)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)^4}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(bx+a\right)^{4}}{\sqrt{\csc\left(bx+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{4}\left(a+bx\right)}{\sqrt{\csc\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(sec(a + b*x)**4/sqrt(csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(bx+a\right)^{4}}{\sqrt{\csc\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

3.279 $\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx$

Optimal. Leaf size=76

$$\frac{d\sqrt{d\cos(a+bx)}\csc^{p-1}(a+bx)\,{}_2F_1\left(-\frac{1}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)\sqrt[4]{\cos^2(a+bx)}}$$

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[-1/4, (1 -
p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*(Cos[a + b*x]^2)^(1/4))
```

Rubi [A] time = 0.105468, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2587, 2577}

$$\frac{d\sqrt{d\cos(a+bx)}\csc^{p-1}(a+bx)\,{}_2F_1\left(-\frac{1}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)\sqrt[4]{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^p,x]
```

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[-1/4, (1 -
p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*(Cos[a + b*x]^2)^(1/4))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx &= \left(\csc^p(a+bx)\sin^p(a+bx)\right)\int (d\cos(a+bx))^{3/2}\sin^{-p}(a+bx)\,dx\\ &= \frac{d\sqrt{d\cos(a+bx)}\csc^{-1+p}(a+bx)\,{}_2F_1\left(-\frac{1}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)\sqrt[4]{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.64128, size = 105, normalized size = 1.38

$$-\frac{2(d\cos(a+bx))^{5/2}\sin^2(a+bx)^{\frac{p-1}{2}}\csc^{p-1}(a+bx)\left(5\cos^2(a+bx)\,{}_2F_1\left(\frac{9}{4},\frac{p+1}{2};\frac{13}{4};\cos^2(a+bx)\right)+9\,{}_2F_1\left(\frac{5}{4},\frac{p-1}{2};\frac{9}{4};\cos^2(a\right.\right.}{45bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^p,x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^(-1 + p)*(9*Hypergeometric2F1[5/4,
(-1 + p)/2, 9/4, Cos[a + b*x]^2] + 5*Cos[a + b*x]^2*Hypergeometric2F1[9/4,
(1 + p)/2, 13/4, Cos[a + b*x]^2])*(Sin[a + b*x]^2)^((-1 + p)/2))/(45*b*d)
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}} (\csc(bx+a))^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x)
```

```
[Out] int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}} \csc(bx+a)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\cos(bx+a)}d\csc(bx+a)^{p}\cos(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d*csc(b*x + a)^p*cos(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^p, x)
```

3.280 $\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx$

Optimal. Leaf size=76

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)\sqrt{d\cos(a+bx)}}$$

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[1/4, (1 -
 p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.091963, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2587, 2577}

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^p,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[1/4, (1 -
 p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*Sqrt[d*Cos[a + b*x]])
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx &= \left(\csc^p(a+bx)\sin^p(a+bx)\right)\int \sqrt{d\cos(a+bx)}\sin^{-p}(a+bx)\,dx\\ &= \frac{d\sqrt[4]{\cos^2(a+bx)}\csc^{-1+p}(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.171021, size = 70, normalized size = 0.92

$$-\frac{2(d\cos(a+bx))^{3/2}\sin^2(a+bx)^{\frac{p-1}{2}}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{p+1}{2};\frac{7}{4};\cos^2(a+bx)\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^p,x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[3/4, (1
+ p)/2, 7/4, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((-1 + p)/2))/(3*b*d)
```

Maple [F] time = 0.31, size = 0, normalized size = 0.

$$\int \sqrt{d \cos \left(b x+a\right)}\, \left(\csc \left(b x+a\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x)
```

```
[Out] int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \cos \left(b x+a\right)} \csc \left(b x+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d \cos \left(b x+a\right)} \csc \left(b x+a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \cos \left(a+b x\right)} \csc^{p}\left(a+b x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**p,x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*csc(a + b*x)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cos(bx+a)}\csc(bx+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^p, x)
```

3.281 $\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=76

$$\frac{d\cos^2(a+bx)^{3/4}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)(d\cos(a+bx))^{3/2}}$$

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[3/4, (1 -
 p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.0973584, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2587, 2577}

$$\frac{d\cos^2(a+bx)^{3/4}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^p/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[3/4, (1 -
 p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*(d*Cos[a + b*x])^(3/2))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx &= \left(\csc^p(a+bx)\sin^p(a+bx)\right)\int \frac{\sin^{-p}(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx\\ &= \frac{d\cos^2(a+bx)^{3/4}\csc^{-1+p}(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.216115, size = 68, normalized size = 0.89

$$-\frac{2\sqrt{d\cos(a+bx)}\sin^2(a+bx)^{\frac{p+1}{2}}\csc^{p+1}(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{p+1}{2};\frac{5}{4};\cos^2(a+bx)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^p/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-2*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^(1 + p)*Hypergeometric2F1[1/4, (1 + p
)/2, 5/4, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 + p)/2))/(b*d)
```

Maple [F] time = 0.299, size = 0, normalized size = 0.

$$\int (\csc (bx+a))^{p} \frac{1}{\sqrt{d\cos (bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] int(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a)^{p}}{\sqrt{d\cos (bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^p/sqrt(d*cos(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{d\cos (bx+a)}\csc (bx+a)^{p}}{d\cos (bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p/(d*cos(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{p}(a+bx)}{\sqrt{d\cos (a+bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**p/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**p/sqrt(d*cos(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a)^p}{\sqrt{d\cos (bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^p/sqrt(d*cos(b*x + a)), x)
```

3.282 $\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt[4]{\cos^2(a+bx)}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{5}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{bd(1-p)\sqrt{d\cos(a+bx)}}$$

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[5/4, (1 - p
)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*d*(1 - p)*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] time = 0.111089, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2587, 2577}

$$\frac{\sqrt[4]{\cos^2(a+bx)}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{5}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{bd(1-p)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^p/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[5/4, (1 - p
)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*d*(1 - p)*Sqrt[d*Cos[a + b*x]])
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= \left(\csc^p(a+bx)\sin^p(a+bx)\right)\int \frac{\sin^{-p}(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx\\ &= \frac{\sqrt[4]{\cos^2(a+bx)}\csc^{-1+p}(a+bx)\,{}_2F_1\left(\frac{5}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{bd(1-p)\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.274321, size = 68, normalized size = 0.87

$$\frac{2\sin^2(a+bx)^{\frac{p-1}{2}}\csc^{p-1}(a+bx)\,{}_2F_1\left(-\frac{1}{4},\frac{p+1}{2};\frac{3}{4};\cos^2(a+bx)\right)}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^p/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[-1/4, (1 + p)/2, 3/4, Cos[a + b*
x]^2]*(Sin[a + b*x]^2)^((-1 + p)/2))/(b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int (\csc (bx + a))^p (d \cos (bx + a))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] int(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx + a)^p}{(d \cos (bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \cos (bx + a)} \csc (bx + a)^p}{d^2 \cos (bx + a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p/(d^2*cos(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**p/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^p}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(3/2), x)
```

3.283 $\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\cos^2(a+bx)^{3/4}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{7}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{bd(1-p)(d\cos(a+bx))^{3/2}}$$

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[7/4, (1 - p
)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*d*(1 - p)*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] time = 0.110108, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2587, 2577}

$$\frac{\cos^2(a+bx)^{3/4}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{7}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{bd(1-p)(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^p/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[7/4, (1 - p
)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*d*(1 - p)*(d*Cos[a + b*x])^(3/2))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx &= \left(\csc^p(a+bx)\sin^p(a+bx)\right)\int \frac{\sin^{-p}(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx\\ &= \frac{\cos^2(a+bx)^{3/4}\csc^{-1+p}(a+bx)\,{}_2F_1\left(\frac{7}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{bd(1-p)(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.286083, size = 70, normalized size = 0.9

$$\frac{2\sin^2(a+bx)^{\frac{p-1}{2}}\csc^{p-1}(a+bx)\,{}_2F_1\left(-\frac{3}{4},\frac{p+1}{2};\frac{1}{4};\cos^2(a+bx)\right)}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^p/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[-3/4, (1 + p)/2, 1/4, Cos[a + b*
x]^2]*(Sin[a + b*x]^2)^((-1 + p)/2))/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int (\csc (bx+a))^{p} (d\cos (bx+a))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] int(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a)^{p}}{(d\cos (bx+a))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\cos (bx+a)}\csc (bx+a)^{p}}{d^{3}\cos (bx+a)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p/(d^3*cos(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**p/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^p}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(5/2), x)
```

3.284 $\int \cos^m(e+fx)\csc^n(e+fx)\,dx$

Optimal. Leaf size=85

$$\frac{\cos^{m-1}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{n-1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] (Cos[e + f*x]^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Csc[e + f*x]^(-1 + n)*H
ypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 -
n))
```

Rubi [A] time = 0.0825669, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2587, 2577}

$$\frac{\cos^{m-1}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{n-1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^m*Csc[e + f*x]^n,x]
```

```
[Out] (Cos[e + f*x]^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Csc[e + f*x]^(-1 + n)*H
ypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 -
n))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx = \left(\csc^n(e+fx)\sin^n(e+fx)\right)\int \cos^m(e+fx)\sin^{-n}(e+fx)\,dx$$
$$= \frac{\cos^{-1+m}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Mathematica [C] time = 1.88532, size = 312, normalized size = 3.67

$$-\frac{2(n-3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)}{f(n-1)\left(2\sin^2\left(\frac{1}{2}(e+fx)\right)\left(mF_1\left(\frac{3}{2}-\frac{n}{2};1-m,m-n+1;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m-n+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^m*Csc[e + f*x]^n,x]
```

```
[Out] (-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Cos[e + f*x]^m*Csc[e + f*x]^n*S
in[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3
/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(
m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n, 5/2 - n/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F] time = 0.927, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{m} \left(\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^m*csc(f*x+e)^n,x)
```

```
[Out] int(cos(f*x+e)^m*csc(f*x+e)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)^{m} \csc\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*csc(f*x+e)^n,x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^m*csc(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos\left(fx+e\right)^{m} \csc\left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*csc(f*x+e)^n,x, algorithm="fricas")
```

```
[Out] integral(cos(f*x + e)^m*csc(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^{m}\left(e+fx\right) \csc^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**m*csc(f*x+e)**n,x)
```

```
[Out] Integral(cos(e + f*x)**m*csc(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)^{m}\csc\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*csc(f*x+e)^n,x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^m*csc(f*x + e)^n, x)
```

3.285 $\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{a\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{n-1}(e+fx)(a\cos(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] (a*(a*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Csc[e + f*x]^(-1
+ n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/(f
*(1 - n))
```

Rubi [A] time = 0.0912086, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2587, 2577}

$$\frac{a\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{n-1}(e+fx)(a\cos(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*Csc[e + f*x]^n,x]
```

```
[Out] (a*(a*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Csc[e + f*x]^(-1
+ n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/(f
*(1 - n))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a\cos(e+fx))^m\csc^n(e+fx)\,dx = \left(\csc^n(e+fx)\sin^n(e+fx)\right)\int (a\cos(e+fx))^m\sin^{-n}(e+fx)\,dx$$

$$= \frac{a(a\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+f\right.}{f(1-n)}$$

Mathematica [C] time = 0.300098, size = 314, normalized size = 3.57

$$-\frac{2(n-3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\csc^{\prime}}{f(n-1)\left(2\sin^2\left(\frac{1}{2}(e+fx)\right)\left(mF_1\left(\frac{3}{2}-\frac{n}{2};1-m,m-n+1;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m-n+1)F\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cos[e + f*x])^m*Csc[e + f*x]^n,x]
```

```
[Out] (-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(a*Cos[e + f*x])^m*Csc[e + f*x]
^n*Sin[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m -
n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 +
 2*(m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n, 5/2 -
 n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F] time = 1.004, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\left(\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*csc(f*x+e)^n,x)
```

```
[Out] int((a*cos(f*x+e))^m*csc(f*x+e)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\csc\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*csc(f*x+e)^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m*csc(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\cos\left(fx+e\right)\right)^{m}\csc\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*csc(f*x+e)^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(f*x + e))^m*csc(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(e+fx\right)\right)^{m}\csc^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*csc(f*x+e)**n,x)
```

```
[Out] Integral((a*cos(e + f*x))**m*csc(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\csc\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*csc(f*x+e)^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*csc(f*x + e)^n, x)
```

3.286 $\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=88

$$\frac{b\cos^{m-1}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] (b*Cos[e + f*x]^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f*x])^(-1
+ n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/(f
*(1 - n))
```

Rubi [A] time = 0.0921135, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2587, 2577}

$$\frac{b\cos^{m-1}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^m*(b*Csc[e + f*x])^n,x]
```

```
[Out] (b*Cos[e + f*x]^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f*x])^(-1
+ n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/(f
*(1 - n))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx = \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \cos^m(e+fx)(b\sin(e+fx))^{-n}\,dx$$
$$= \frac{b\cos^{-1+m}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e\right.}{f(1-n)}$$

Mathematica [C] time = 0.270629, size = 314, normalized size = 3.57

$$-\frac{2(n-3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\,c}{f(n-1)\left(2\sin^2\left(\frac{1}{2}(e+fx)\right)\left(mF_1\left(\frac{3}{2}-\frac{n}{2};1-m,m-n+1;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m-n+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^m*(b*Csc[e + f*x])^n,x]
```

```
[Out] (-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Cos[e + f*x]^m*(b*Csc[e + f*x])
^n*Sin[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m -
n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 +
 2*(m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n, 5/2 -
 n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F] time = 0.98, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{m} \left(b \csc \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^m*(b*csc(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^m*(b*csc(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{n} \cos \left(fx+e\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc \left(fx+e\right)\right)^{n} \cos \left(fx+e\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*cos(f*x + e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(e+fx\right)\right)^{n} \cos^{m} \left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**m*(b*csc(f*x+e))**n,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*cos(e + f*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\cos\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^m, x)
```

3.287 $\int (a\cos(e+fx))^m (b\csc(e+fx))^n\, dx$

Optimal. Leaf size=91

$$\frac{ab\cos^2(e+fx)^{\frac{1-m}{2}}(a\cos(e+fx))^{m-1}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] (a*b*(a*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f*x]
)^(-1 + n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^
2])/(f*(1 - n))
```

Rubi [A] time = 0.0987331, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2587, 2577}

$$\frac{ab\cos^2(e+fx)^{\frac{1-m}{2}}(a\cos(e+fx))^{m-1}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^n,x]
```

```
[Out] (a*b*(a*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f*x]
)^(-1 + n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^
2])/(f*(1 - n))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx = \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int (a\cos(e+fx))^m(b\sin(e+fx))^{-n}\,dx$$

$$= \frac{ab(a\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin\ldots\right.}{f(1-n)}$$

Mathematica [C] time = 0.275486, size = 316, normalized size = 3.47

$$-\frac{2(n-3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(a\cos\ldots}{f(n-1)\left(2\sin^2\left(\frac{1}{2}(e+fx)\right)\left(mF_1\left(\frac{3}{2}-\frac{n}{2};1-m,m-n+1;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m-n+1)F\ldots\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^n,x]
```

```
[Out] (-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(a*Cos[e + f*x])^m*(b*Csc[e + f
*x])^n*Sin[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 +
m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]
^2 + 2*(m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n, 5
/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F] time = 1.109, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\left(b\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\left(b\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*csc(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\cos\left(fx+e\right)\right)^{m}\left(b\csc\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(f*x + e))^m*(b*csc(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(e+fx\right)\right)^{m}\left(b\csc\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**n,x)
```

```
[Out] Integral((a*cos(e + f*x))**m*(b*csc(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\left(b\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*csc(f*x + e))^n, x)
```

3.288 $\int (a\cos(e+fx))^m (b\csc(e+fx))^{7/2}\,dx$

Optimal. Leaf size=78

$$-\frac{b^3\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{9}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{af(m+1)}$$

```
[Out] -((b^3*(a*Cos[e + f*x])^(1 + m)*Sqrt[b*Csc[e + f*x]]*Hypergeometric2F1[9/4,
 (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(1/4))/(a*f*(1 + m)
))
```

Rubi [A] time = 0.117567, antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{b\sin^2(e+fx)^{5/4}(b\csc(e+fx))^{5/2}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{9}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(7/2),x]
```

```
[Out] -((b*(a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(5/2)*Hypergeometric2F1[9/4,
 (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(5/4))/(a*f*(1 + m)
))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int (a\cos(e+fx))^m(b\csc(e+fx))^{7/2}\,dx &= \left(b^2(b\csc(e+fx))^{5/2}(b\sin(e+fx))^{5/2}\right)\int\frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{7/2}}\,dx\\ &= -\frac{b(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{5/2}\,{}_2F_1\left(\frac{9}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin^2(}{af(1+m)}\end{aligned}$$

Mathematica [A] time = 7.90656, size = 94, normalized size = 1.21

$$\frac{2ab(b\csc(e+fx))^{5/2}\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}(a\cos(e+fx))^{m-1}\,{}_2F_1\left(\frac{1}{4}(7-2m),\frac{1-m}{2};\frac{1}{4}(11-2m);\csc^2(e+fx)\right)}{f(2m-7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(7/2),x]
```

```
[Out] (2*a*b*(a*Cos[e + f*x])^(-1 + m)*(-Cot[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f
*x])^(5/2)*Hypergeometric2F1[(7 - 2*m)/4, (1 - m)/2, (11 - 2*m)/4, Csc[e +
f*x]^2])/(f*(-7 + 2*m))
```

Maple [F] time = 0.361, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m} \left(b\csc\left(fx+e\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{\frac{7}{2}} \left(a\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^(7/2)*(a*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\csc\left(fx+e\right)}\left(a\cos\left(fx+e\right)\right)^{m} b^{3}\csc\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m*b^3*csc(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{\frac{7}{2}}\left(a\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^(7/2)*(a*cos(f*x + e))^m, x)
```

3.289 $\int (a\cos(e+fx))^m(b\csc(e+fx))^{5/2}\,dx$

Optimal. Leaf size=76

$$-\frac{b\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{af(m+1)}$$

```
[Out] -((b*(a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(3/2)*Hypergeometric2F1[7/4,
 (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(3/4))/(a*f*(1 + m)
))
```

Rubi [A] time = 0.115817, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{b\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(5/2),x]
```

```
[Out] -((b*(a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(3/2)*Hypergeometric2F1[7/4,
 (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(3/4))/(a*f*(1 + m)
))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int (a\cos(e+fx))^m(b\csc(e+fx))^{5/2}\,dx &= \left(b^2(b\csc(e+fx))^{3/2}(b\sin(e+fx))^{3/2}\right)\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{5/2}}\,dx\\ &= -\frac{b(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{3/2}\,{}_2F_1\left(\frac{7}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin^2(e+}{af(1+m)}\end{aligned}$$

Mathematica [A] time = 2.45064, size = 94, normalized size = 1.24

$$\frac{2ab(b\csc(e+fx))^{3/2}\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}(a\cos(e+fx))^{m-1}\,{}_2F_1\left(\frac{1}{4}(5-2m),\frac{1-m}{2};\frac{1}{4}(9-2m);\csc^2(e+fx)\right)}{f(2m-5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(5/2),x]
```

```
[Out] (2*a*b*(a*Cos[e + f*x])^(-1 + m)*(-Cot[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f
*x])^(3/2)*Hypergeometric2F1[(5 - 2*m)/4, (1 - m)/2, (9 - 2*m)/4, Csc[e + f
*x]^2])/(f*(-5 + 2*m))
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\left(b\csc\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{\frac{5}{2}}\left(a\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^(5/2)*(a*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\csc\left(fx+e\right)}\left(a\cos\left(fx+e\right)\right)^{m}b^{2}\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m*b^2*csc(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{\frac{5}{2}}\left(a\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^(5/2)*(a*cos(f*x + e))^m, x)
```

3.290 $\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx$

Optimal. Leaf size=76

$$-\frac{b\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{af(m+1)}$$

```
[Out] -((b*(a*Cos[e + f*x])^(1 + m)*Sqrt[b*Csc[e + f*x]]*Hypergeometric2F1[5/4, (
1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(1/4))/(a*f*(1 + m)))
```

Rubi [A] time = 0.112016, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{b\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(3/2),x]
```

```
[Out] -((b*(a*Cos[e + f*x])^(1 + m)*Sqrt[b*Csc[e + f*x]]*Hypergeometric2F1[5/4, (
1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(1/4))/(a*f*(1 + m)))
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx &= \left(b^2\sqrt{b\csc(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{3/2}}\,dx\\ &= -\frac{b(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,{}_2F_1\left(\frac{5}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sqrt[4]{\sin^2(}}{af(1+m)}\end{aligned}$$

Mathematica [A] time = 1.25679, size = 94, normalized size = 1.24

$$\frac{2ab\sqrt{b\csc(e+fx)}\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}(a\cos(e+fx))^{m-1}\,{}_2F_1\left(\frac{1}{4}(3-2m),\frac{1-m}{2};\frac{1}{4}(7-2m);\csc^2(e+fx)\right)}{f(2m-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(3/2),x]
```

```
[Out] (2*a*b*(a*Cos[e + f*x])^(-1 + m)*(-Cot[e + f*x]^2)^((1 - m)/2)*Sqrt[b*Csc[e
 + f*x]]*Hypergeometric2F1[(3 - 2*m)/4, (1 - m)/2, (7 - 2*m)/4, Csc[e + f*x
]^2])/(f*(-3 + 2*m))
```

Maple [F] time = 0.33, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\left(b\csc\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^(3/2)*(a*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\csc\left(fx+e\right)}\left(a\cos\left(fx+e\right)\right)^{m}b\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m*b*csc(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^(3/2)*(a*cos(f*x + e))^m, x)
```

3.291 $\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx$

Optimal. Leaf size=78

$$-\frac{\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)}$$

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(3/2)*Hypergeometric2F1[3/4, (
1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(3/4))/(a*b*f*(1 + m)
))
```

Rubi [A] time = 0.0984332, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2586, 2576}

$$-\frac{\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*Sqrt[b*Csc[e + f*x]],x]
```


```
[Out] -(((a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(3/2)*Hypergeometric2F1[3/4, (
1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(3/4))/(a*b*f*(1 + m)
))
```


Rule 2586

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n + 1))
/b^2, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx &= \frac{\left((b\csc(e+fx))^{3/2}(b\sin(e+fx))^{3/2}\right)\int \frac{(a\cos(e+fx))^m}{\sqrt{b\sin(e+fx)}}\,dx}{b^2}\\ &= -\frac{(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin^2(e+fx)}{abf(1+m)}\end{aligned}$$

Mathematica [A] time = 1.08033, size = 96, normalized size = 1.23

$$\frac{2\tan(e+fx)\sqrt{b\csc(e+fx)}\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}(a\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{4}(1-2m),\frac{1-m}{2};\frac{1}{4}(5-2m);\csc^2(e+fx)\right)}{f(2m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*Sqrt[b*Csc[e + f*x]],x]
```

```
[Out] (2*(a*Cos[e + f*x])^m*(-Cot[e + f*x]^2)^((1 - m)/2)*Sqrt[b*Csc[e + f*x]]*Hy
pergeometric2F1[(1 - 2*m)/4, (1 - m)/2, (5 - 2*m)/4, Csc[e + f*x]^2]*Tan[e
+ f*x])/(f*(-1 + 2*m))
```

Maple [F] time = 0.374, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\sqrt{b\csc\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\csc\left(fx+e\right)}\left(a\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\csc\left(fx+e\right)}\left(a\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(e+fx\right)\right)^{m}\sqrt{b\csc\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*cos(e + f*x))**m*sqrt(b*csc(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \csc(fx+e)}\,(a\cos(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m, x)
```

3.292 $\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)}$$

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Sqrt[b*Csc[e + f*x]]*Hypergeometric2F1[1/4, (1
+ m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(1/4))/(a*b*f*(1 + m)))
```

Rubi [A] time = 0.097994, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2586, 2576}

$$-\frac{\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m/Sqrt[b*Csc[e + f*x]],x]
```

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Sqrt[b*Csc[e + f*x]]*Hypergeometric2F1[1/4, (1
+ m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(1/4))/(a*b*f*(1 + m)))
```

Rule 2586

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n + 1))
/b^2, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx &= \frac{\left(\sqrt{b\csc(e+fx)}\sqrt{b\sin(e+fx)}\right)\int (a\cos(e+fx))^m\sqrt{b\sin(e+fx)}\,dx}{b^2}\\ &= -\frac{(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sqrt[4]{\sin^2(e+fx)}}{abf(1+m)}\end{aligned}$$

Mathematica [C] time = 1.71423, size = 225, normalized size = 2.88

$$\frac{14b(a\cos(e+fx))^mF_1\left(\frac{3}{4};-m,m+\right.}{3f(b\csc(e+fx))^{3/2}\left(7F_1\left(\frac{3}{4};-m,m+\frac{3}{2};\frac{7}{4};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan^2\left(\frac{1}{2}(e+fx)\right)\left(2mF_1\left(\frac{7}{4};1-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cos[e + f*x])^m/Sqrt[b*Csc[e + f*x]],x]
```

```
[Out] (14*b*AppellF1[3/4, -m, 3/2 + m, 7/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(a*Cos[e + f*x])^m)/(3*f*(b*Csc[e + f*x])^(3/2)*(7*AppellF1[3/4, -m, 3/
2 + m, 7/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(2*m*AppellF1[7/4,
 1 - m, 3/2 + m, 11/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)
*AppellF1[7/4, -m, 5/2 + m, 11/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2))
```

Maple [F] time = 0.365, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m} \frac{1}{\sqrt{b\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x)
```

```
[Out] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(fx+e\right)\right)^{m}}{\sqrt{b\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m/sqrt(b*csc(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\csc\left(fx+e\right)}\left(a\cos\left(fx+e\right)\right)^{m}}{b\csc\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m/(b*csc(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(e+fx\right)\right)^{m}}{\sqrt{b\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m/(b*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*cos(e + f*x))**m/sqrt(b*csc(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(fx+e\right)\right)^m}{\sqrt{b\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m/sqrt(b*csc(f*x + e)), x)
```

3.293 $\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{(a\cos(e+fx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}}$$

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, C
os[e + f*x]^2])/(a*b*f*(1 + m)*Sqrt[b*Csc[e + f*x]]*(Sin[e + f*x]^2)^(1/4))
)
```

Rubi [A] time = 0.11167, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2586, 2576}

$$-\frac{(a\cos(e+fx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(3/2),x]
```

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, C
os[e + f*x]^2])/(a*b*f*(1 + m)*Sqrt[b*Csc[e + f*x]]*(Sin[e + f*x]^2)^(1/4))
)
```

Rule 2586

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n + 1))
/b^2, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx &= \frac{\int (a\cos(e+fx))^m(b\sin(e+fx))^{3/2}\,dx}{b^2\sqrt{b\csc(e+fx)}\sqrt{b\sin(e+fx)}}\\ &= -\frac{(a\cos(e+fx))^{1+m}\,{}_2F_1\left(-\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)}{abf(1+m)\sqrt{b\csc(e+fx)}\sqrt[4]{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 6.76986, size = 116, normalized size = 1.49

$$\frac{2a\cos(2(e+fx))\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}(a\cos(e+fx))^{m-1}\,{}_2F_1\left(\frac{1}{4}(-2m-3),\frac{1-m}{2};\frac{1}{4}(1-2m);\csc^2(e+fx)\right)}{bf(2m+3)\left(\csc^2(e+fx)-2\right)\sqrt{b\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(3/2),x]
```

```
[Out] (2*a*(a*Cos[e + f*x])^(-1 + m)*Cos[2*(e + f*x)]*(-Cot[e + f*x]^2)^((1 - m)/
2)*Hypergeometric2F1[(-3 - 2*m)/4, (1 - m)/2, (1 - 2*m)/4, Csc[e + f*x]^2])
/(b*f*(3 + 2*m)*Sqrt[b*Csc[e + f*x]]*(-2 + Csc[e + f*x]^2))
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^m \left(b\csc\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x)
```

```
[Out] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(fx+e\right)\right)^m}{\left(b\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\csc\left(fx+e\right)}\left(a\cos\left(fx+e\right)\right)^m}{b^2\csc\left(fx+e\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m/(b^2*csc(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m/(b*csc(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(fx+e))^m}{(b\csc(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(3/2), x)
```

3.294 $\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(-\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{ab^3f(m+1)}$$

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Sqrt[b*Csc[e + f*x]]*Hypergeometric2F1[-3/4, (1
 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(1/4))/(a*b^3*f*(1 + m
)))
```

Rubi [A] time = 0.111243, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2586, 2576}

$$-\frac{(a\cos(e+fx))^{m+1}\,{}_2F_1\left(-\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(5/2),x]
```

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[-3/4, (1 + m)/2, (3 + m)/2, C
os[e + f*x]^2])/(a*b*f*(1 + m)*(b*Csc[e + f*x])^(3/2)*(Sin[e + f*x]^2)^(3/4
)))
```

Rule 2586

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n + 1))
/b^2, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx &= \frac{\int (a\cos(e+fx))^m(b\sin(e+fx))^{5/2}\,dx}{b^2(b\csc(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}\\ &= -\frac{(a\cos(e+fx))^{1+m}\,{}_2F_1\left(-\frac{3}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)}{abf(1+m)(b\csc(e+fx))^{3/2}\sin^2(e+fx)^{3/4}}\end{aligned}$$

Mathematica [A] time = 1.11356, size = 125, normalized size = 1.6

$$\frac{2(2\cos(2(e+fx))+1)\tan(e+fx)\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}(a\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{4}(-2m-5),\frac{1-m}{2};\frac{1}{4}(-2m-1);\csc^2(e+\right.}{b^2f(2m+5)\left(3\csc^2(e+fx)-4\right)\sqrt{b\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(5/2),x]
```

```
[Out] (2*(a*Cos[e + f*x])^m*(1 + 2*Cos[2*(e + f*x)])*(-Cot[e + f*x]^2)^((1 - m)/2
)*Hypergeometric2F1[(-5 - 2*m)/4, (1 - m)/2, (-1 - 2*m)/4, Csc[e + f*x]^2]*
Tan[e + f*x])/(b^2*f*(5 + 2*m)*Sqrt[b*Csc[e + f*x]]*(-4 + 3*Csc[e + f*x]^2)
)
```

Maple [F] time = 0.318, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\left(b\csc\left(fx+e\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x)
```

```
[Out] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(fx+e\right)\right)^{m}}{\left(b\csc\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\csc\left(fx+e\right)}\left(a\cos\left(fx+e\right)\right)^{m}}{b^{3}\csc\left(fx+e\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m/(b^3*csc(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m/(b*csc(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(fx+e))^m}{(b\csc(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(5/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
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                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

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```python
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
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        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```